Lecture Notes · Unit 1 · Lecture 14 AP CED 1.13

Removing discontinuities

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INTRO: FIX A BREAK WITH ONE VALUE?

Removing Discontinuities AP Calculus 1.13

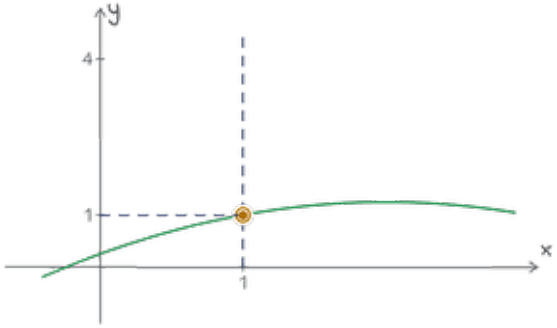
Break at c → fix it with one value?

1 value · find k · no fix

IDEA 1: MOVE THE DOT INTO THE HOLE

Key ideas

1. Move the dot



New value = limit

In symbols $f(1) = \lim_{x \to 1} f(x) = 1$

Limit still 1

$\lim_{x \to 1} f(x) = 1 = f(1)$ → continuous at 1

Hole? Set f(c) = limit

1 value → removable (1.10)

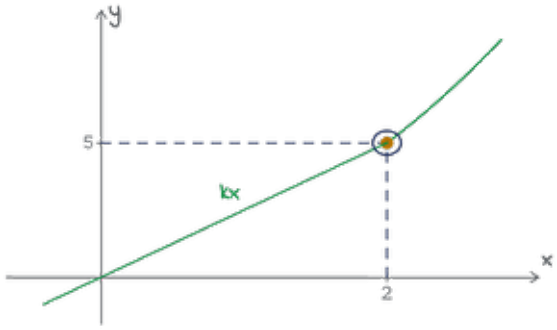
IDEA 2: FIND K

Key ideas

2. Find k

find k so f is continuous

$f(x) = kx\ (x < 2),\ x^2 + 1\ (x \geq 2)$



(1) the = sign in $x \geq 2 \rightarrow f(2) = 2^2 + 1 = 5$

$\lim_{x \to 2^+} (x^2 + 1) = 5,\ \lim_{x \to 2^-} kx = 2k$

Continuous: left = right = f(2)

$2k = 5 \rightarrow k = \frac{5}{2}$

$\frac{5}{2} \cdot 2 = 5 = f(2)$

AP: left = right = f(2) → then solve

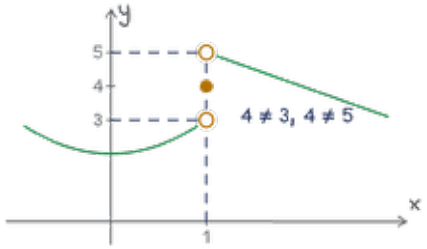
IDEA 3: IF POSSIBLE

Key ideas

3. If possible: can one value fix a jump?

if possible

e.g. $x^2 + 2\ (x < 1),\ 6 - x\ (x > 1)$



$\lim_{x \to 1^-} (x^2 + 2) = 1 + 2 = 3$

$\lim_{x \to 1^+} (6 - x) = 6 - 1 = 5$

$3 \neq 5$ → no limit

other f(1)?

f(1) can't help → limit ignores it

(ii) fails, any f(1) → no value works

Asymptote → ±∞ → no fix

A k can fail 1 + k = k + 3 → 1 = 3

Fixable ⇔ limit exists

PROBLEM 1 OF 3

f(2) = k

Let $f(x)=\begin{cases}\dfrac{2x^2-x-6}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$

Find the value of k that makes f continuous at $x=2$.

Solution The limit first, then k equals it

Plan: $f(2)=\lim_{x\to 2} f(x)$

→ find the limit, then k = that limit

$x \neq 2$ & $-2-6=0 \to \frac{0}{0}$ → factor

$$\lim_{x\to 2}\frac{(2x+3)(x-2)}{(x-2)}=\lim_{x\to 2}(2x+3)=7$$

(i) f(2) = k

(ii) $\lim_{x\to 2} f(x) = 7$

(iii) they match if k = 7

→ f is continuous at 2

Workbook → limit first, then k

PROBLEM 2 OF 3

g(0) = m

Let $g(x)=\begin{cases}\dfrac{3\sin x}{x}, & x\neq 0\\ m, & x=0\end{cases}$

Find the value of m that makes g continuous at $x=0$.

Special limit (1.8)

$$\lim_{\square\to 0}\frac{\sin\square}{\square}=1$$

Solution The limit first, using the special limit

Plan: $g(0)=\lim_{x\to 0} g(x)$ → find it, then m

$$\lim_{x\to 0} g(x)=3\cdot\lim_{x\to 0}\frac{\sin x}{x}$$

$= 3 \cdot 1$ (1.8) $= 3$

(i) g(0) = m (ii) limit 3 (iii) m = 3

m = 3 → continuous at 0 → hole (0, 3) filled

Workbook → special limit (1.8)

PROBLEM 3 OF 3

Two constants at two boundaries

Let $f(x)=\begin{cases}ax^2-3, & x<-1\\ 2x+b, & -1\le x\le 2\\ ax, & x>2\end{cases}$.

For what values of a and b is f continuous at $x=-1$ and at $x=2$?

(A) $a=1,\ b=-2$

(B) $a=-1,\ b=-6$

(C) $a=3,\ b=2$

(D) $a=2,\ b=0$

Continuous at a boundary c

$$\lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)=f(c)$$

Solution

One equation at each boundary, solved together

Plan: a, b → 2 equations

Solve as a pair

Middle includes −1, 2 → 2x + b

$$\lim_{x \to -1^-} \left(ax^2 - 3\right) = a(-1)^2 - 3 = a - 3$$

$$\lim_{x \to -1^+} (2x + b) = f(-1) = -2 + b$$

$$a - 3 = -2 + b \rightarrow b = a - 1$$

$$\lim_{x \to 2^-} (2x + b) = f(2) = 4 + b$$

$$\lim_{x \to 2^+} ax = a(2) = 2a$$

$$4 + b = 2a \rightarrow b = 2a - 4$$

$$a - 1 = 2a - 4 \rightarrow a = 3 \; b = 3 - 1 = 2$$

So (C) → a = 3, b = 2

Workbook: boundary → 1 eq.

WRAP-UP

Fix a break? Sometimes

Limit exists → set f(c) = the limit

Boundary → left = right = f(c) → solve

Then (i), (ii), (iii) → conclusion last

Jump, asymptote → no limit → no value works

Next → 1.14 → infinite limits

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.