1.13 / Removing Discontinuities

Watch the lesson ➤

Removing discontinuities

Learning goal

Remove a discontinuity when it is possible — define or redefine one value, or choose constants at the boundaries of a piecewise function — and explain with limits when it is not possible.

Key ideas

01 A break at c: can we fix it? The limit is 1, but $f(1) = 4$: redefine $f(1) = \lim_{x\to 1} f(x) = 1$. Now $\lim_{x\to 1} f(x) = f(1)$: continuous at 1. No value yet? Define $f(c) =$ the limit. One value fixes it: **removable**.

02 Find k: $f(x) = kx\,(x < 2), x^2{+}1\,(x \geq 2)$. (i) $x \geq 2$ has the $=$: $f(2) = 2^2{+}1 = 5$. $\lim_{x\to 2^+}(x^2 + 1) = 5$, $\lim_{x\to 2^-} kx = 2k$. Continuous $\iff$ left = right = $f(2)$: $2k = 5$, $k = \frac{5}{2}$.

03 Two constants, two equations: for $4x + 1\ (x < 1)$, $x^2 + 4\ (1 \leq x \leq 2)$, $4x\ (x > 2)$, at 1: $4(1) + 1 = 5 = 1^2 + 4$; at 2: $2^2 + 4 = 8 = 4(2)$. Each boundary gives one equation. Hide two of the numbers as a and b: two unknowns, both boundaries, two equations. Solve them **together**.

04 If possible: $x^2 + 2\ (x < 1)$, $6 - x\ (x > 1)$, $f(1) = 4$. The sides head to 3 and 5; $3 \neq 5$, so there is no limit, and whatever $f(1)$ is, no value works. An asymptote is unbounded: no value works. A k can fail too: $1 + k = k + 3$ gives $1 = 3$. A value fixes it $\iff$ the limit **exists**.

Remember

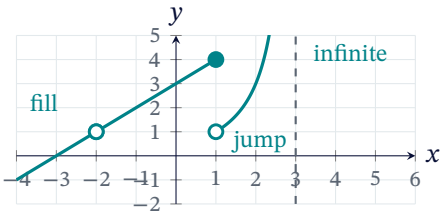
removable discontinuity
The limit exists, but $f(c)$ is missing or different. One new value fixes it.

redefine
Give $f(c)$ a new value, here the value of the limit.

boundary
An x-value where a piecewise function changes from one piece to another.

constant
A letter such as k, a, or b that stands for one fixed number.

One value can fill a hole, but not a jump or an asymptote



At $x = -2$ both sides head to 1: define $f(-2) = 1$. At $x = 1$ the sides head to 4 and 1, and at $x = 3$ the sides are unbounded: the limit does not exist, so no value works.

Class notes

1.13 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show both one-sided limits and the value with numbers, then conclude: "$\lim_{x\to 2^-} f(x) = 11$, $\lim_{x\to 2^+} f(x) = 11$, and $f(2) = 11$. So f is continuous at $x = 2$."	Do not write "$k = 3$, so f is continuous" without the values that meet the conditions.
Keep lim on every line until you substitute: $\lim_{x\to 4} \frac{(x+7)(x-4)}{x-4} = \lim_{x\to 4}(x+7) = 11$.	Do not write a limit equal to $\frac{0}{0}$. Write "direct substitution gives the 0/0 form," then rewrite.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The value at the boundary must match too

Code L11

✗ $f(x) = x^2 - k$ for $x < 3$, $f(3) = 2$, $f(x) = x + k$ for $x > 3$. "$9 - k = 3 + k$, so $k = 3$, and f is continuous at $x = 3$."

✓ With $k = 3$ both sides head to 6, but $f(3) = 2$. All three must be equal, so no value of k works.

02 0/0 means rewrite, not give up

Code L1

✗ $f(x) = \dfrac{x^2 - x - 30}{x - 6}$. "Direct substitution gives 0/0, so the discontinuity at $x = 6$ cannot be removed."

✓ Factor: $\lim_{x\to 6} \frac{(x+5)(x-6)}{x-6} = \lim_{x\to 6}(x+5) = 11$. The limit exists, so define $f(6) = 11$.

03 Check both sides before you redefine

Code L3

✗ $\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = 5$. "Define $f(1) = 2$ to remove the discontinuity."

✓ The one-sided limits differ, so the limit does not exist: a jump. No value of $f(1)$ removes it.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks to remove a discontinuity or to choose a constant for continuity, and no released multiple-choice question in our sources does either. The items follow the CED learning objective LIM-2.C (values or constants that make a function continuous, if possible): LIM-2.C.1 (redefine the value as the limit) and LIM-2.C.2 (both sides and the value agree at a boundary).

1.13 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Redefine $f(c)$ at a removable discontinuity	Find $\lim_{x\to c} f(x)$ (factor, rationalize, or match the angle after the 0/0 form). Define $f(c)$ as that limit.	E01, E02 · Practice 1, 6
A removable discontinuity read from a graph or a table	Read the height both sides approach. The new value of $f(c)$ is that height, not the filled dot.	Practice 2
One constant at a boundary of a piecewise function	Left piece at c = right piece at c ($= f(c)$). Solve for the constant, then check with values.	
A constant and a separate value at the boundary	All three must be equal: left piece at c, right piece at c, and the given or unknown $f(c)$.	Practice 3
Two constants at two boundaries	Write one equation at each boundary. Solve the two equations together.	E03
If possible: a jump or an infinite discontinuity cannot be removed	Find both one-sided limits. Different real numbers, or an unbounded side: the limit does not exist, so no value of $f(c)$ works.	Practice 4, 5
A constant inside a rational formula	The bottom approaches 0, so the top must too: solve top$(c) = 0$. Then factor, find the limit, and define $f(c)$.	

Example 01

Video · E01

Let $f(x) = \begin{cases} \dfrac{2x^2 - x - 6}{x - 2}, & x \neq 2 \\ k, & x = 2 \end{cases}$

Find the value of k that makes f continuous at $x = 2$.

> For continuity at 2, first find the limit of the formula used near 2.

Example 02

Video · E02

Let $g(x) = \begin{cases} \dfrac{3\sin x}{x}, & x \neq 0 \\ m, & x = 0 \end{cases}$

Find the value of m that makes g continuous at $x = 0$.

> Use the special trigonometric limit from 1.8, with the angle in radians.

Example 03

Video · E03

$$f(x) = \begin{cases} ax^2 - 3, & x < -1 \\ 2x + b, & -1 \leq x \leq 2 \\ ax, & x > 2 \end{cases}$$

For which a, b is f continuous at $x = -1$ and at $x = 2$?

A. $a = 1,\ b = -2$

B. $a = -1,\ b = -6$

C. $a = 3,\ b = 2$

D. $a = 2,\ b = 0$

> Two constants need two equations, one at each boundary.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.13 / Example answers

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E01 7

E02 3

E03 (C)