1.12 / Practice workbook

Continuity over an Interval

Practice

Problems 1–5 / Show your reasoning

1 The functions f and g are continuous for all real numbers, and $g(x) = 0$ only at $x = 3$. Which of the following must be continuous for all real numbers?

I. $f(x)\,g(x)$

II. $\dfrac{f(x)}{g(x)}$

III. $\dfrac{f(x)}{[g(x)]^2 + 1}$

A. I only

B. I and II only

C. I and III only

D. I, II, and III

1.12 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{x+3}{\ln x}$.

(a) Find all values of x at which f is not defined. Give a reason for each.

(b) Find the largest open intervals on which f is continuous. Justify your answer.

1.12 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} \sqrt{4-x}, & x < 0 \\ 2 + \ln(x+1), & x \geq 0 \end{cases}$

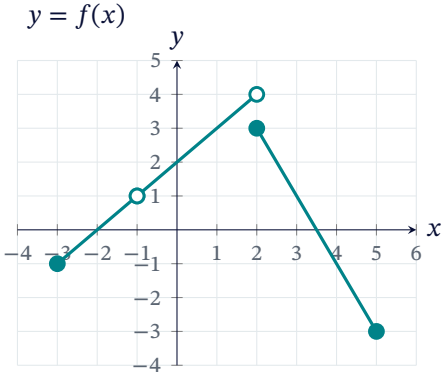
(a) Explain why f is continuous for $x < 0$ and for $x > 0$.

(b) Is f continuous at $x = 0$? Justify your answer.

(c) On what interval is f continuous?

4 The graph of f is shown. On which of the following intervals is f continuous?

$y = f(x)$



A. $[-3, 0]$
B. $[-1, 1]$
C. $[0, 2]$
D. $[2, 5]$

1.12 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{x-9}{\sqrt{x}-3}$. A student says: "$f(x) = \sqrt{x} + 3$, so f is continuous on $[0, \infty)$."

(a) Find $\lim_{x \to 9} f(x)$.

(b) Is the student right? Explain.

(c) Find the largest intervals on which f is continuous.

1.12 / Answer key

Continuity over an Interval

1 (C)

2 (a) $x \leq 0$: $\ln x$ needs $x > 0$. $x = 1$: $\ln 1 = 0$, a zero denominator.; (b) $(0, 1)$ and $(1, \infty)$: $x + 3$ and $\ln x$ are continuous for $x > 0$, so the quotient is continuous wherever $\ln x \neq 0$

3 (a) For $x < 0$: $4 - x > 4 > 0$, so the root is defined and continuous. For $x > 0$: $x + 1 > 1 > 0$, so $\ln(x + 1)$ is defined and continuous.; (b) yes: $f(0) = 2 + \ln 1 = 2$, $\lim_{x \to 0^-} \sqrt{4 - x} = 2$, $\lim_{x \to 0^+}(2 + \ln(x + 1)) = 2$, so the limit is $2 = f(0)$; (c) $(-\infty, \infty)$: every real number

4 (D)

5 (a) 6; (b) no: in the given formula $f(9)$ is undefined ($\sqrt{9} - 3 = 0$), so condition (i) fails and f is not continuous at $x = 9$, even though the limit is 6; simplifying does not fill the hole; (c) $[0, 9)$ and $(9, \infty)$