Lecture Notes · Unit 1 · Lecture 13 AP CED 1.12

Continuity on an interval

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INTRO: CAN A FUNCTION BE CONTINUOUS AT AN END?

Continuity on an Interval AP Calculus 1.12

$\sqrt{x}$ at 0 → nothing on the left → continuous?

where continuous? → interval

End: one side · reason: one line · domain first

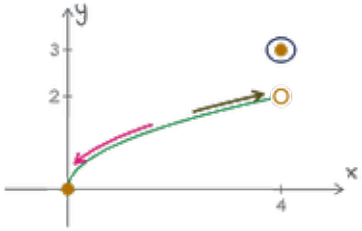
IDEA 1: AT AN ENDPOINT, CHECK ONE SIDE

Key ideas

1. on an interval, check one side at an endpoint

$[a, b]$ ends

e.g. $f(x) = \sqrt{x}$ $(0 \le x < 4)$, $f(4) = 3$



At 0 → (i) $f(0) = 0$

Nothing left of 0

No → (ii) inside only: $x \to 0^+$

$\lim_{x \to 0^+} \sqrt{x} = 0$ (iii) $0 = f(0)$ ok

→ continuous from the right at 0

At 4 → (i) $f(4) = 3$, filled dot

(ii) $\lim_{x \to 4^-} \sqrt{x} = 2$ (iii) limit ≠ value

→ not continuous at 4

IDEA 1: BRACKETS AND THE CLOSED INTERVAL

Key ideas

End in → [; left out →)

Continuous on $[0, 4)$ → closed? No.

on $[a, b]$ ⇔ every x in (a, b)

$\lim_{x \to a^+} f(x) = f(a)$, $\lim_{x \to b^-} f(x) = f(b)$

Ends → inside only → rest: (i)(ii)(iii)

IDEA 2: WHOLE FAMILIES ON THEIR DOMAINS

Key ideas

2. whole families at once

6 families

Polynomial, rational, power (roots), exp, log, trig

continuous on domain → plug in

1.6, 1.8

reason ?

Name the family

f rational → continuous on domain · 1 line for all x

Domain → 4 exclusions

1. bottom = 0
2. even root of a negative
3. ln of 0 or less
4. $\tan x = \frac{\sin x}{\cos x}$

→ out where $\cos x = 0$

IDEA 3: PUT CONTINUOUS FUNCTIONS TOGETHER

Key ideas

3. Continuous at c

f, g continuous at c

$\lim_{x \to c} f(x) = f(c) \quad \lim_{x \to c} g(x) = g(c)$

$\lim_{x \to c} [f(x) + g(x)] = f(c) + g(c)$

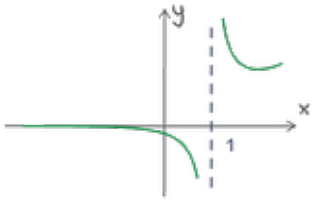
→ f+g continuous at c

Same → $f - g$ $f \cdot g$; $\frac{f}{g}$ if $g(c) \neq 0$

Composite $f(g(x))$ → inside rule (1.11)

e.g. $\frac{e^x}{x-1}$ → top, bottom continuous

$x - 1 = 0$ only at $x = 1$

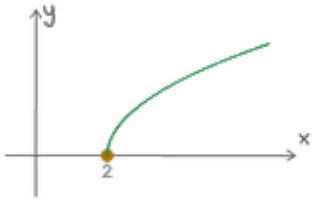


Continuous: $(-\infty, 1)$ and $(1, \infty)$

IDEA 3: A COMPOSITE WITH A CLOSED END

Key ideas

e.g. $\sqrt{x-2}$ → inside ≥ 0 → $x \geq 2$



$[2, \infty)$ continuous · closed at 2

IDEA 4: PIECES AND BOUNDARIES

Key ideas

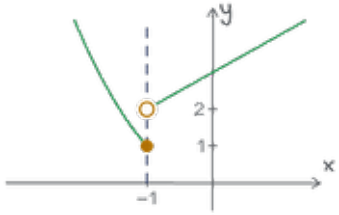
4. A piecewise function

pieces

Pieces → own family own interval

Boundaries: (i), (ii), (iii)

$f(x) = x^2 \ (x \leq -1), \ x + 3 \ (x > -1)$



Polynomials → only $x = -1$ to check

(i) = in $x \leq -1$ → $f(-1) = (-1)^2 = 1$

$\lim_{x \to -1^-} x^2 = 1, \quad \lim_{x \to -1^+} (x+3) = 2$

$1 \neq 2$ → (ii) fails → not continuous at −1

Left = $f(-1)$ → $(-\infty, -1]$ and $(-1, \infty)$

IDEA 5: THE DOMAIN COMES BEFORE YOU CANCEL

Key ideas

5. Find the domain before you cancel!

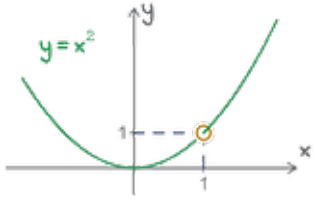
given formula first

e.g. $f(x) = \frac{x^3 - x^2}{x - 1}$

$x = 1$ → bottom $1 - 1 = 0$ → $f(1)$ undefined

Just x^2? $x^3 - x^2 = x^2(x - 1)$

$x \neq 1$ → $\frac{x^2(x-1)}{(x-1)} = x^2$



Given f → $f(1)$ undefined → (i) fails

$x = 1 \rightarrow$ discontinuous

Cancel → a limit, not the domain

Continuous $(-\infty, 1)$ and $(1, \infty)$

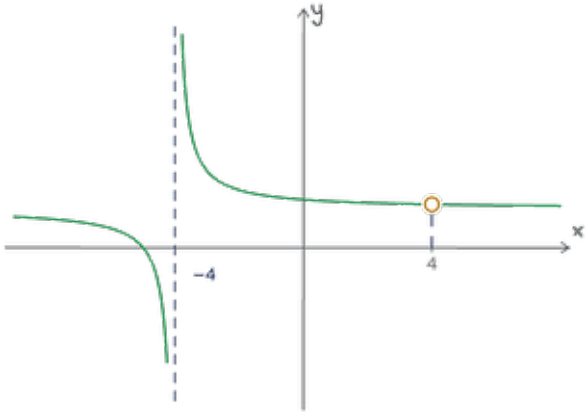
PROBLEM 1 OF 2

Where a rational function is continuous

Let $f(x) = \dfrac{x^2 + x - 20}{x^2 - 16}$

(a) At which values of x is f not continuous? Explain.

(b) Find the largest open intervals on which f is continuous.



Solution

Name the family, then the domain from the given formula

Plan: rational → continuous on domain

Domain of given f

(a)

$x^2 - 16 = 0 \quad \Leftrightarrow \quad (x+4)(x-4) = 0$

$= 0$ at $x = -4$ and $x = 4$

$f(-4)$, $f(4)$ undefined → (i) fails

-4, 4: undefined → not continuous

(b)

Domain: $x \neq -4, 4$

$(-\infty, -4)$, $(-4, 4)$, $(4, \infty)$

f rational: continuous on each interval

Workbook → the given formula

Common mistake

✗ Common mistake

$\dfrac{(x+5)(x-4)}{(x+4)(x-4)} = \dfrac{x+5}{x+4}$

→ only $x = -4$ out

✓ Do this

Given f → f(4) undefined

Hole stays

out: $-4, 4$

PROBLEM 2 OF 2

Continuous on a closed interval

Which of the following functions is continuous on $[0, 3]$?

(A) $f(x) = \sqrt{x - 1}$

(B) $g(x) = \tan x$

(C) $h(x) = \ln x$

(D) $k(x) = \dfrac{e^x}{x + 1}$

Solution

The domain must contain all of the closed interval

Plan: $[0, 3]$, both ends → inside the domain

(D) $x + 1 = 0$ only at $x = -1$, not in $[0, 3]$

Both continuous; bottom $\neq 0$ → quotient

(A) $\sqrt{x-1}$ → needs $x \geq 1$, misses $[0, 1)$ ✗

(B) $\dfrac{\sin x}{\cos x}$: $0 < \dfrac{\pi}{2} \approx 1.57 < 3$ ✗

(C) $\ln x \rightarrow \ln 0$ undefined, 0 is an end ✗

So (D)

Workbook → interval in domain

WRAP-UP



Where is f continuous?

Closed end → from the inside

Name the family → continuous on its domain

Domain → from the given formula

Pieces → each boundary (i)(ii)(iii)

AP → 'continuous on [a, b] because' + rule + domain check

Next → 1.13 → removing discontinuities

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.