1.12 / Continuity over an Interval

Watch the lesson ➤

Confirming continuity over an interval

Learning goal
Find the intervals where a function is continuous from its formula, its pieces, or its graph, and give the reason with the function families.

Key ideas

01 The ends of an interval look one way: $f(x) = \sqrt{x}$ $(0 \le x < 4)$, $f(4) = 3$. At 0: (i) $f(0) = 0$, (ii) $\lim_{x\to 0^+} \sqrt{x} = 0$, (iii) $0 = f(0)$, so f is continuous from the right at 0. At 4: (i) $f(4) = 3$, (ii) $\lim_{x\to 4^-} \sqrt{x} = 2$, (iii) $2 \neq 3$, not continuous at 4. So f is continuous on $[0, 4)$, not on $[0, 4]$.

02 On $[a, b]$: continuous at every x in (a, b), and at the ends $\lim_{x\to a^+} f(x) = f(a)$, $\lim_{x\to b^-} f(x) = f(b)$. The ends: **one side** only.

03 Polynomial, rational, power, root, exponential, logarithmic, trigonometric: continuous on their **domains**, so plug in. On the exam, the reason is the family's name.

04 Put them together: if $\lim_{x\to c} f(x) = f(c)$ and $\lim_{x\to c} g(x) = g(c)$, then $\lim_{x\to c}[f(x) + g(x)] = f(c) + g(c)$, so $f + g$ is continuous at c. The same laws give $f - g$ and $f \cdot g$; f/g too if $g(c) \neq 0$; $f(g(x))$ if g is continuous at c and f at $g(c)$. E.g. $\dfrac{e^x}{x-1}$: the top and the bottom are continuous, and $x - 1 = 0$ only at $x = 1$, so it is continuous on $(-\infty, 1)$ and $(1, \infty)$. Pieces: each its own family; boundaries: (i), (ii), (iii).

05 Why an x drops out of a domain: a bottom $= 0$, an even root of a negative, ln of 0 or less, $\tan x = \dfrac{\sin x}{\cos x}$ where $\cos x = 0$. E.g. $\sqrt{x-2}$ needs $x - 2 \ge 0$, so $x \ge 2$: a composite, continuous on $[2, \infty)$, from inside at 2.

06 The given formula, before cancelling: $f(x) = \dfrac{x^3 - x^2}{x - 1}$. At $x = 1$ the bottom is $1 - 1 = 0$, so $f(1)$ is undefined. For $x \neq 1$, $f(x) = x^2$: the graph is $y = x^2$ with a hole at $(1, 1)$, and (i) fails, so f is not continuous at $x = 1$. Cancelling gives a limit, not the **domain**: f is continuous on $(-\infty, 1)$ and $(1, \infty)$.

Remember

continuous on an interval
Continuous at every x in the interval; at a closed endpoint, the side inside counts.

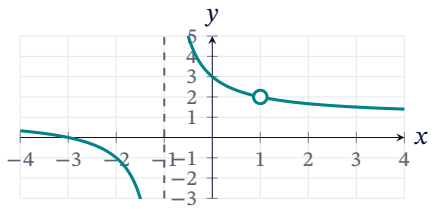
domain
All x where the formula gives a real number.

interval notation
(a, b) leaves out the endpoints, $[a, b]$ keeps them, and $\cup$ joins intervals.

endpoint
A number at either end of an interval.

The domain decides where f is continuous

$\sqrt{u}$: $u \ge 0$ $\quad$ $\ln u$: $u > 0$
$\tan x$, $\sec x$: $\cos x \neq 0$ $\left(x \neq \frac{\pi}{2} + k\pi\right)$
$\arcsin u$: $-1 \le u \le 1$
e^x, $\sqrt[3]{x}$, $\arctan x$, $\sin x$, $\cos x$: all real x



$f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 1}$ is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$: a vertical asymptote at $x = -1$ and a hole at $(1, 2)$.

Class notes

1.12 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Give the reason with the family and the domain: "f is a rational function and its denominator is not 0 on $[1, 3]$, so f is continuous on $[1, 3]$."	Do not write "f is continuous" without the reason.
Name each excluded x with its reason: "$f(4)$ is undefined because the denominator is 0, so f is not continuous at $x = 4$."	Do not write a limit equal to $\frac{0}{0}$ when you check a boundary. Write "direct substitution gives the 0/0 form," then rewrite.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Cancelling does not fill the hole

Code L2

✗ $f(x) = \dfrac{x^2 - x - 6}{x^2 - 9} = \dfrac{x + 2}{x + 3}$, "so f is continuous on $(-\infty, -3) \cup (-3, \infty)$."

✓ $f(3)$ is undefined in the given formula. f is continuous on $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$.

02 Polynomial pieces still need a boundary check

Code L10

✗ $f(x) = x^2 + 2$ for $x < 1$ and $f(x) = 2x$ for $x \geq 1$. "Both pieces are polynomials, so f is continuous everywhere."

✓ At $x = 1$ the one-sided limits are 3 and 2, so (ii) fails. f is continuous on $(-\infty, 1) \cup (1, \infty)$.

03 Flip the sign when you divide by a negative

Code L4

✗ $\sqrt{10 - 5x}$: "$10 - 5x \geq 0$, so $x \geq 2$."

✓ $-5x \geq -10$; dividing by -5 flips the sign: $x \leq 2$. The root is continuous on $(-\infty, 2]$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks for the intervals on which a function is continuous. Free-response questions on Topic 1.16 need a stated reason that a function is continuous on a closed interval, the sentence practiced in this lesson.

1.12 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Where a formula is continuous (find the domain)	Name the family. Find the domain: no zero denominator, an even root of a number ≥ 0, ln of a number > 0, $\cos x \neq 0$ for $\tan x$ and $\sec x$, $-1 \leq u \leq 1$ for $\arcsin u$. f is continuous on its domain.	E01 · Practice 2, 5
Where a piecewise function is continuous	Inside each piece use its family; drop excluded points that lie in that piece. At each boundary check (i), (ii), (iii). Join the intervals.	Practice 3
Is f continuous on a given interval?	No break inside the interval. At a closed endpoint, the one-sided limit from inside must equal the value.	Practice 4
Intervals of continuity from a graph	Mark every hole, jump, asymptote, and moved point. The intervals between the marks are the answer.	
Give the reason for continuity on an interval	Name the family and show that no excluded point lies in the interval: "f is rational and its denominator is not 0 on $[a, b]$, so f is continuous on $[a, b]$."	
Which function is continuous on an interval or everywhere?	For each function, find its domain and compare it with the interval.	E02 · Practice 1

Example 01

Video · E01

Let $f(x) = \dfrac{x^2 + x - 20}{x^2 - 16}$.

Find the domain before you cancel.

(a) Find all x where f is not continuous. Give a reason.

(b) List the largest open intervals where f is continuous. Use interval notation.

Example 02

Video · E02

Which function is continuous on $[0, 3]$?

A. $f(x) = \sqrt{x - 1}$

B. $g(x) = \tan x$

C. $h(x) = \ln x$

D. $k(x) = \dfrac{e^x}{x + 1}$

> Find each domain, then look at the whole interval, endpoints too.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.12 / Example answers

Continuity over an Interval

E01 (a) $x = -4$ and $x = 4$: $f(-4)$ and $f(4)$ are undefined (the denominator is 0), so condition (i) fails; (b) $(-\infty, -4)$, $(-4, 4)$, and $(4, \infty)$: f is rational, so it is continuous on its domain

E02 (D)