1.11 / Practice workbook

Continuity at a Point

Practice

Problems 1–7 / Show your reasoning

1 The function f is continuous at $x = -1$, and $f(-1) = 4$. Which of the following must be true?

I. $\lim_{x \to -1^+} f(x) = 4$

II. $\lim_{x \to -1} [f(x) - 4] = 0$

III. $\lim_{x \to -1} \dfrac{f(x)}{x + 1}$ exists.

A. I only

B. II only

C. I and II only

D. I, II, and III

1.11 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} \dfrac{x^2 - 9}{|x| - 3}, & 0 < x < 3 \\ 2x, & x \geq 3 \end{cases}$

(a) Find $\lim_{x \to 3^-} f(x)$.

(b) Is f continuous at $x = 3$? Justify your answer using the definition of continuity.

1.11 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Selected values of f are given in the table.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	2.61	2.9601	2.996	3	3.004	3.0401	3.41

(a) Use the table to estimate $\lim_{x \to 2} f(x)$.

(b) Which conditions for continuity does the table confirm?

(c) Does the table alone prove continuity at $x = 2$?

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Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} x^2 + x + 2, & x \le 1 \\ 7 - 2x, & x > 1 \end{cases}$

(a) Find $\lim_{x \to 1^-} f(x)$.

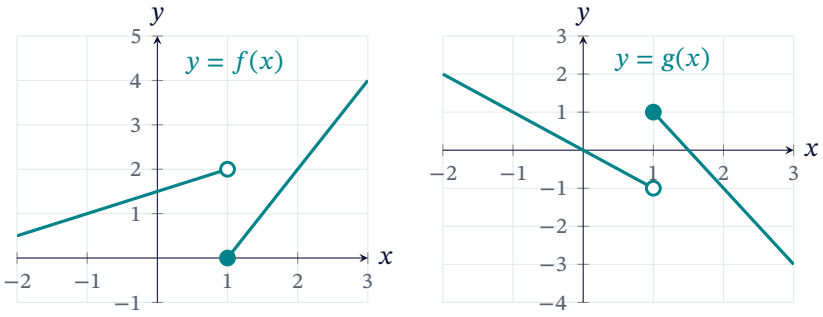
(b) Find $\lim_{x \to 1^+} f(x)$.

(c) Is f continuous at $x = 1$? Explain.

1.11 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

The graphs of f and g are shown. Neither function is continuous at $x = 1$. Let $p(x) = f(x) + g(x)$.



(a) Find $\lim_{x \to 1^-} p(x)$ and $\lim_{x \to 1^+} p(x)$.

(b) Is p continuous at $x = 1$? Justify your answer.

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Let $f(x) = \begin{cases} \dfrac{\sqrt{x+12}-4}{x-4}, & x \geq -12,\ x \neq 4 \\ \dfrac{1}{8}, & x = 4 \end{cases}$

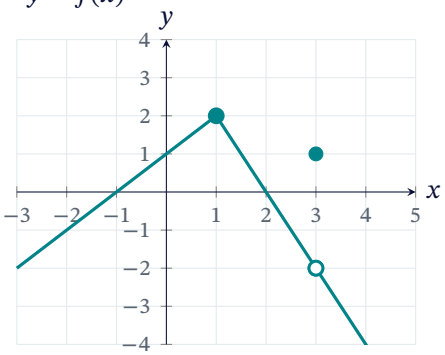
Which of the following is true about f at $x = 4$?

A. f is continuous at $x = 4$.
B. Condition (i) fails: $f(4)$ is undefined.
C. Condition (ii) fails: the limit does not exist.
D. Condition (iii) fails: $\lim_{x \to 4} f(x) \neq f(4)$.

7

The graph of f is shown. Which of the following statements are true?
I. f is continuous at $x = 1$.
II. $\lim_{x \to 3} f(x)$ exists.
III. f is continuous at $x = 3$.

$y = f(x)$



A. I only
B. I and II only
C. I and III only
D. I, II, and III

1.11 / Answer key

Continuity at a Point

1 (C)

2 (a) 6; (b) yes: (i) $f(3) = 2(3) = 6$, (ii) both one-sided limits are 6, so the limit is 6, (iii) the limit equals $f(3)$

3 (a) 3: the values approach 3 from both sides; (b) (i) is confirmed: the table gives $f(2) = 3$. (ii) and (iii) are only suggested: a table cannot prove that the limit is 3.; (c) No: the table is consistent with continuity, but between its rows f could jump or oscillate, so the limit is not proved.

4 (a) 4; (b) 5; (c) no: (i) $f(1) = 4$ is defined, but (ii) $\lim_{x \to 1^-} f(x) = 4$ and $\lim_{x \to 1^+} f(x) = 5$ differ, so $\lim_{x \to 1} f(x)$ does not exist

5 (a) both are 1: $2 + (-1) = 1$ from the left and $0 + 1 = 1$ from the right; (b) yes: $p(1) = 0 + 1 = 1$, both one-sided limits are 1, and the limit equals $p(1)$

6 (A)

7 (B)