Lecture Notes · Unit 1 · Lecture 12 AP CED 1.11

Continuity at a point

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INTRO: NO BREAK, IN WORDS

Continuity at a Point AP Calculus 1.11

Breaks have names (1.10)

What does no break mean?

Eyes see it → a grader reads words

Three checks + one equation

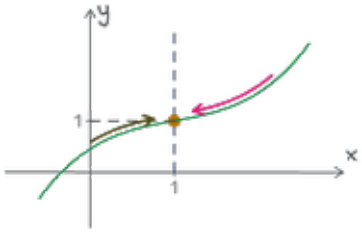
IDEA 1: CONTINUITY AT A POINT

Key ideas

No break at c → prove it in words?

1. No break at x = 1 → what's true there?

no break at x = 1



(i) a filled dot → f(1) = 1

(ii) both sides → 1 → $\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 1} f(x) = f(1)$?

(iii) limit = value → 1 = 1

No break → f is continuous at x = 1

Continuous at c → 3 checks

(i) f(c) is defined

(ii) $\lim_{x \to c} f(x)$ exists

(iii) $\lim_{x \to c} f(x) = f(c)$

ONE EQUATION, AND A CONTINUOUS FUNCTION

Key ideas

$\lim_{x \to c} f(x) = f(c) \Rightarrow$ continuous at x = c

Equal → both sides exist → all three

Whole domain → continuous function

x = 1: ✓ all 3

→ not a continuous function

Every point → continuous f

Continuous function → every x in its domain

Continuity on the domain

$f(x) = \frac{1}{x}$ 0 not in the domain

Continuous for $x \neq 0$ → continuous function

Read it → 'at x = 1' (a point) or 'on [a, b]'

The limit goes inside

$\lim_{x \to a} x = a$

$f(a) = f\left(\lim_{x \to a} x\right)$ (a in the domain) same a

Continuous at a → $\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to a} f(x) = f\left(\lim_{x \to a} x\right)$

Plug in works → that's continuity

THE LIMIT GOES INSIDE: COMPOSITES

Key ideas

$g(x) \to L$, f continuous at L → $\lim_{x \to a} f(g(x)) = f(L)$

e^x no breaks → $\lim_{x \to 0} e^{\sin x} = e^{\lim_{x \to 0} \sin x} = e^0 = 1$

ln inside → 1 ⇒ $\lim_{x\to 2} \ln\left(x^2-3\right) = \ln 1 = 0$

$\lim_{x\to 3} \sqrt{x+6} = \sqrt{9} = 3$

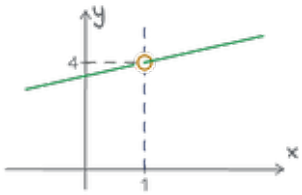
Inside limit? outside continuous there?

IDEA 2: WHY ALL THREE CONDITIONS

Key ideas

2. Why all three?

3 graphs

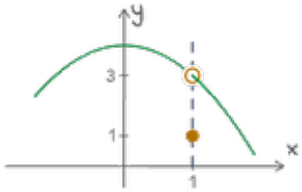


(ii) ✓ limit 4; (i) fails

no value → removable

(i) ✓ f(1) = 3; (ii) fails

3 ≠ 1 → a jump



(i) ✓, (ii) ✓; (iii) fails

3 ≠ 1 → removable

(i) + (ii) + (iii) · if one fails → name it

Exam → (i), (ii), (iii) first · conclusion last

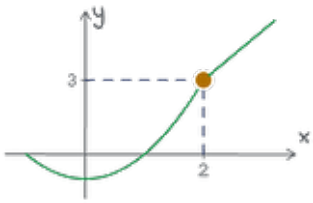
IDEA 3: A BOUNDARY

Key ideas

3. Where two formulas meet

seam → three checks

e.g. $f(x) = x^2 - 1\ (x < 2),\ 2x - 1\ (x \geq 2)$



(i) the = sign in $x \geq 2$ → $f(2) = 2(2) - 1 = 3$

(ii) $\lim_{x\to 2^-} \left(x^2-1\right) = 3$, $\lim_{x\to 2^+} (2x-1) = 3$ → 3

(iii) $\lim_{x\to 2} f(x) = 3 = f(2) = 3$ ✓

So f is continuous at 2 → conditions first

→ $f(3) = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

PROBLEM 1 OF 2

A formula and a separate value

Let $f(x) = \begin{cases} \dfrac{x^2+x-12}{x-3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$

(a) Find $\lim_{x\to 3} f(x)$.

(b) Is f continuous at $x = 3$? Justify your answer.

Continuous at c

(i) f(c) defined

(ii) $\lim_{x\to c} f(x)$ exists

(iii) $\lim_{x\to c} f(x) = f(c)$

Solution Find the limit, then check the three conditions

Plan: (a) the limit → (b) (i), (ii), (iii)

(a)

$x \neq 3 \rightarrow f(x) = \dfrac{x^2+x-12}{x-3}$

$\dfrac{0}{0} \rightarrow x^2+x-12 = (x-3)(x+4)$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{(x-3)(x+4)}{(x-3)} = \lim_{x\to 3} (x+4) = 7$$

(b)

(i) $x = 3 \rightarrow f(3) = 4 \rightarrow$ defined

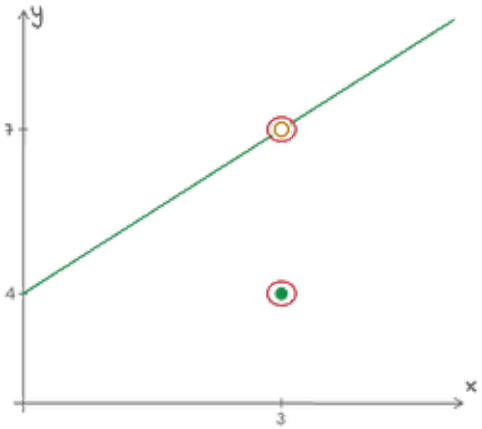
(ii) $\lim_{x\to 3} f(x) = 7 \rightarrow$ exists

(iii) $7 \neq 4 = f(3) \rightarrow$ (iii) fails

So f is not continuous at x = 3, (iii) fails

Workbook → all three, in order

Careful → a common slip



✗ Common mistake

$f(3) = 4$

→ defined

→ continuous

only (i) checked

✓ Do this

Check all three

$\lim_{x \to 3} f(x) = 7$

$7 \neq 4 = f(3)$ → (iii) fails

PROBLEM 2 OF 2

Boundaries of a piecewise function

Let $f(x) = \begin{cases} 2 - 3\sin x, & x < 0 \\ x + 2, & 0 \le x \le 2 \\ 3e^{x-2}, & x > 2 \end{cases}$

(a) Is f continuous at $x = 0$? Justify your answer.

(b) Is f continuous at $x = 2$? Justify your answer.

Solution

The value from the piece with =, each side from its own piece, then compare

Plan: (i) value from its piece

(ii) sides, own piece (iii) compare

(a)

(i) middle $0 \le x \le 2$ → $f(0) = 0 + 2 = 2$

(ii) $\lim_{x \to 0^-} (2 - 3\sin x) = 2 - 3\sin 0 = 2$

$\lim_{x \to 0^+} (x + 2) = 2$

Both sides 2 → $\lim_{x \to 0} f(x) = 2$

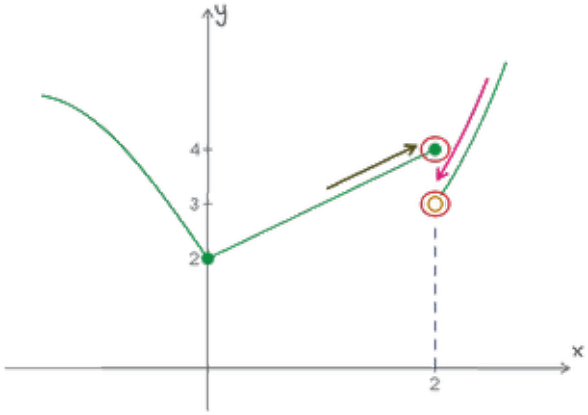
(iii) $\lim_{x \to 0} f(x) = 2 = f(0)$

All 3 hold → so f is continuous at $x = 0$

Workbook → value, sides, compare

Solution

The value from the piece with =, each side from its own piece, then compare



Plan: the same three, at 2

(b)

(i) middle $0 \le x \le 2$ → $f(2) = 2 + 2 = 4$

(ii) $\lim_{x \to 2^-} (x + 2) = 4$

$\lim_{x \to 2^+} 3e^{x-2} = 3e^0 = 3$

$4 \neq 3$ → no limit → (ii) fails

So f is not continuous at x = 2, (ii) fails

Workbook → (i), (ii) or (iii)?

WRAP-UP

No break? Prove it

f(c) defined, limit exists, equal → continuous

one fails → not continuous, name it

Conditions first → conclusion last

Every point of the domain → a continuous function

Continuous → the limit goes inside

AP → $f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

Next → 1.12: continuity on an interval

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.