1.11 / Continuity at a Point

Watch the lesson ➤

Defining continuity at a point

Learning goal

Decide whether a function is continuous at a point from a formula, a piecewise function, a graph, or a table, and justify it with the three conditions.

Key ideas

01 No break at c: how do we prove it? At $x = 1$ with no break: (i) a filled dot, $f(1) = 1$; (ii) both sides head to 1, $\lim_{x\to 1} f(x) = 1$; (iii) limit = value, $1 = 1$. So f is **continuous** at $x = 1$. In general, f is continuous at c if (i) $f(c)$ is defined, (ii) $\lim_{x\to c} f(x)$ exists, and (iii) $\lim_{x\to c} f(x) = f(c)$.

02 One equation: $\lim_{x\to c} f(x) = f(c)$; with $x = c + h$, $\lim_{h\to 0} f(c + h) = f(c)$. The same at every x: $\lim_{h\to 0} f(x+h) = f(x)$, i.e. $\lim_{h\to 0}[f(x+h)-f(x)] = 0$. Continuous at every x: a continuous **function**.

03 A point or the whole function? A graph with one jump, at $x = 3$, is continuous at $x = 1$ (all three hold there), but it is **not** a continuous function. A graph with no break is continuous at every point: a continuous function. A continuous function means continuous at every point of its **domain**: $\frac{1}{x}$ is a continuous function, since $x = 0$ is not in its domain, yet it is not continuous at $x = 0$. Answer what is asked: "f is continuous at $x = 1$" (a point) or "f is continuous on $[a, b]$" (an interval).

04 The limit goes inside: $\lim_{x\to a} x = a$, so $f(a) = f(\lim_{x\to a} x)$. If f is continuous at a, $\lim_{x\to a} f(x) = f(\lim_{x\to a} x)$. If $g(x) \to L$ and f is continuous at L, then $\lim f(g(x)) = f(L)$: $\lim_{x\to 0} e^{\sin x} = e^0 = 1$, $\lim_{x\to 2} \ln(x^2 - 3) = \ln 1 = 0$, $\lim_{x\to 3} \sqrt{x+6} = \sqrt{9} = 3$. Check first: the inside's limit, and the outside continuous there.

05 Why all three: a hole at $x = 1$ has limit 4 but no value: (i) fails. Left $\to 3$, right $\to 1$ with $f(1) = 3$: (ii) fails. A hole and a dot, limit 3 but $f(1) = 1$: (iii) fails. Continuous $\iff$ (i), (ii), (iii) all hold; one fails, not continuous: name it. On the exam, (i), (ii), (iii) first, the conclusion **last**.

06 A boundary: $f(x) = x^2-1\,(x < 2), 2x-1\,(x \geq 2)$. (i) $x \geq 2$ has the $=$: $f(2) = 3$. (ii) $\lim_{x\to 2^-}(x^2 - 1) = 3$, $\lim_{x\to 2^+}(2x - 1) = 3$. (iii) $\lim_{x\to 2} f(x) = 3 = f(2)$. Therefore f is **continuous** at $x = 2$.

Remember

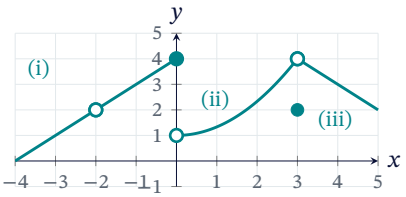
continuous at $x = c$
$f(c)$ is defined, the limit exists, and the limit equals $f(c)$.

condition
Something you must check before you conclude. Continuity at a point has three.

justify
Give the reason: check each condition with values, then write the conclusion.

real number
A number on the number line. ∞ is not a real number.

One condition fails at each break



At $x = -2$ there is no dot: (i) fails. At $x = 0$ the sides head to 4 and 1: (ii) fails. At $x = 3$ the limit is 4 but $f(3) = 2$: (iii) fails.

Class notes

1.11 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Check each condition with values, then conclude: "(i) $f(2) = 5$. (ii) $\lim_{x\to 2^-} f(x) = 5$ and $\lim_{x\to 2^+} f(x) = 5$, so $\lim_{x\to 2} f(x) = 5$. (iii) $\lim_{x\to 2} f(x) = f(2)$. So f is continuous at $x = 2$."	Do not write "f is continuous" without the reason. Show the values that meet the conditions.
Keep lim on every line until you substitute: $\lim_{x\to 3} \frac{(x+4)(x-3)}{x-3} = \lim_{x\to 3}(x+4) = 7$.	Do not write a limit equal to $\frac{0}{0}$. Write "direct substitution gives the 0/0 form," then rewrite.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Defined is not enough

Code L10

✗ $f(x) = \dfrac{x^2 - 4x - 12}{x - 6}$ for $x \neq 6$, and $f(6) = 2$. "$f(6)$ is defined, so f is continuous at $x = 6$."

✓ Also find the limit: $\lim_{x\to 6}(x + 2) = 8$. $8 \neq 2$, so (iii) fails: f is not continuous at $x = 6$.

02 Check both sides

Code L3

✗ $\lim_{x\to 1^-} f(x) = 4$ and $f(1) = 4$, so f is continuous at $x = 1$.

✓ Also find the right-hand limit. If it is 2, the sides differ, the limit does not exist, and (ii) fails.

03 A missing value is condition (i)

Code L2

✗ $f(2)$ is undefined, so $\lim_{x\to 2} f(x)$ does not exist.

✓ The limit depends on x near 2, not at 2. If both sides head to 3, the limit is 3. Condition (i) fails, not (ii).

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks for continuity at a point by the definition. The 2012 BC Practice Exam multiple-choice question 83 asks what must be true when f is continuous at a point, and the 2012 Course Description AB question 17 reads where a limit exists but f is not continuous on a graph.

1.11 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
A formula with a separate value at $x = c$	(i) Read $f(c)$ from the line $x = c$. (ii) Find the limit with the formula for $x \neq c$ (after the 0/0 form: factor, rationalize, or match the angle of a special trig limit). (iii) Compare.	E01 · Practice 6
A boundary of a piecewise function	(i) $f(c)$ from the piece that includes $x = c$. (ii) Each one-sided limit with its own piece; they must be the same real number. (iii) Compare with $f(c)$.	E02 · Practice 2, 4
Continuity read from a graph	(i) Is $f(c)$ defined? Read it from a filled dot or from a point on the curve; an open circle alone means undefined. (ii) Do both sides head to the same height? (iii) Is $f(c)$ that height?	Practice 5, 7
What must be true when f is continuous at $x = c$	Continuity gives all three at once: $f(c)$ exists, both one-sided limits exist, and both equal $f(c)$.	Practice 1
Name the condition that fails	Check in order: (i), then (ii), then (iii). The first "no" is the reason.	
What a table suggests about continuity	Estimate the limit from both sides of the table, then compare with the given $f(c)$. A table only suggests.	Practice 3

Example 01

Video · E01

Let

$$f(x) = \begin{cases} \dfrac{x^2 + x - 12}{x - 3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$$

A defined value is only condition (i).

(a) Find $\lim_{x \to 3} f(x)$.

(b) Is f continuous at $x = 3$? Justify your answer.

Example 02

Video · E02

Let

$$f(x) = \begin{cases} 2 - 3\sin x, & x < 0 \\ x + 2, & 0 \le x \le 2 \\ 3e^{x-2}, & x > 2 \end{cases}$$

(i) the piece with the equals sign; (ii) each side with its own piece; (iii) compare.

(a) Is f continuous at $x = 0$? Justify your answer.

(b) Is f continuous at $x = 2$? Justify your answer.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.11 / Example answers

Continuity at a Point

E01 (a) 7; (b) no: (i) $f(3) = 4$ is defined; (ii) $\lim_{x \to 3} f(x) = 7$ exists; (iii) $7 \neq 4 = f(3)$, so condition (iii) fails

E02 (a) yes: (i) $f(0) = 2$; (ii) both one-sided limits are 2, so $\lim_{x \to 0} f(x) = 2$; (iii) the limit equals $f(0)$; (b) no: $f(2) = 4$, but $\lim_{x \to 2^-} f(x) = 4$ and $\lim_{x \to 2^+} f(x) = 3$ differ, so the limit does not exist; condition (ii) fails