1.10 / Practice workbook

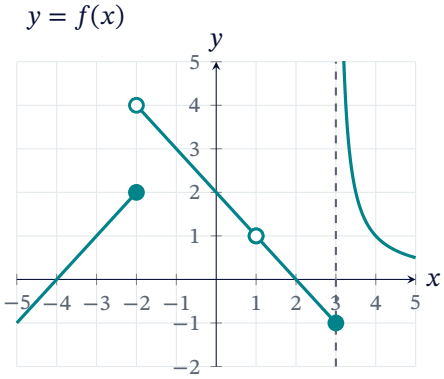
Types of Discontinuities

Practice

Problems 1–5 / Show your reasoning

1 The graph of f is shown. Name the types of discontinuity at $x = -2$, 1, and 3, in that order.

$y = f(x)$



A. jump, removable, jump
B. jump, removable, infinite
C. removable, removable, infinite
D. jump, infinite, infinite

2 Which of the following functions has a removable discontinuity at $x = 4$?

A. $f(x) = \dfrac{x+4}{x^2-16}$
B. $g(x) = \dfrac{x^2-16}{x^2-8x+16}$
C. $h(x) = \dfrac{x^2-5x+4}{x^2-16}$
D. $p(x) = \dfrac{|x-4|}{x-4}$

1.10 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Sketch the graph of one function f with all of the following:
a removable discontinuity at $x = -1$, with $f(-1) = 3$;
a jump discontinuity at $x = 2$, with $\lim_{x \to 2^-} f(x) = f(2) = 1$;
an infinite discontinuity at $x = 4$, with $\lim_{x \to 4^-} f(x) = 2$.

1.10 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+2}, & x < 0 \\ \dfrac{x+6}{x-3}, & x \geq 0 \end{cases}$. The function f has discontinuities at $x = -2$, $x = 0$, and $x = 3$.

(a) Name the type of discontinuity at $x = -2$. Give a reason.

(b) Name the type of discontinuity at $x = 0$. Give a reason.

(c) Name the type of discontinuity at $x = 3$. Give a reason.

5

Selected values of g are given in the table, and $f(x) = \dfrac{2x^2 + 3x + 1}{x + 1}$. Which of the following are true?

I. g appears to have a jump discontinuity at $x = -1$.
II. f has a removable discontinuity at $x = -1$.
III. $\lim_{x \to -1^+} f(x)g(x)$ appears to be -2.

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
$g(x)$	1.9	1.99	1.999	2	4.001	4.01	4.1

A. I only
B. II only
C. I and II only
D. I, II, and III

1.10 / Answer key

Types of Discontinuities

1 (B)

2 (C)

3 One example: $f(x) = \dfrac{(x-2)^2}{3} + 1$ for $x \le 2$, $x \neq -1$, with $f(-1) = 3$; $f(x) = 4 - \dfrac{x}{2}$ for $2 < x < 4$; $f(x) = \dfrac{1}{x-4}$ for $x > 4$

4 (a) removable: $x+2$ cancels, the limit is -1, and $f(-2)$ is undefined; (b) jump: $\lim_{x \to 0^-} f(x) = 1$ and $\lim_{x \to 0^+} f(x) = -2$, two different real numbers; (c) infinite: $x - 3$ stays in the bottom; as $xo3$ the top approaches 9 and the bottom approaches 0, so $f(x)$ is unbounded

5 (C)