Lecture Notes · Unit 1 · Lecture 11 AP CED 1.10

Types of discontinuities

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: NAME THE BREAK

Types of Discontinuities AP Calculus 1.10

A graph breaks → every break the same?

1 graph → 3 breaks

3 names · 1 question

Fraction → hole or wall?

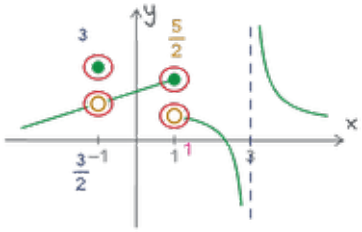
IDEA 1: NAME THE BREAK, THE SIDES FIRST

Key ideas

1. Name the break

both sides first

Ask: both sides → the same real number?



At -1 → both $\frac{3}{2}$, but $f(-1)=3$ → removable

$x=1$ → left $\frac{5}{2}$ ≠ right 1 (both real) → jump

At 3 → a side → $\pm\infty$ → infinite

1.8: $\frac{\sin x}{x}$ at 0 → removable

1.3: $\sin\frac{1}{x}$ near 0 → oscillation

Name alone → not a reason

Limits → name

$\lim_{x\to -1} f(x)=\frac{3}{2}\neq f(-1)$ → removable

$\lim_{x\to 3^-} f(x)=-\infty$ → vertical asymptote

IDEA 2: ZERO IN THE BOTTOM, HOLE OR ASYMPTOTE

Key ideas

2. Zero in the bottom: hole or asymptote?

bottom = 0 → ?

e.g. $f(x)=\frac{x^2-4}{x^2-x-2}$

Bottom = 0 → $x=2, x=-1$

Two zeros → two asymptotes?

Not yet: factor $f(x)=\frac{(x-2)(x+2)}{(x-2)(x+1)}$

$x\to 2, x\neq 2 \to x-2\neq 0$

$f(x)=\frac{(x-2)(x+2)}{(x-2)(x+1)}$

$=\frac{x+2}{x+1}$ $(x\neq 2)$

$\lim_{x\to 2} f(x)=\frac{2+2}{2+1}=\frac{4}{3}$

$f(2)=$?

Limit $\frac{4}{3}$, $f(2)$ undefined → removable

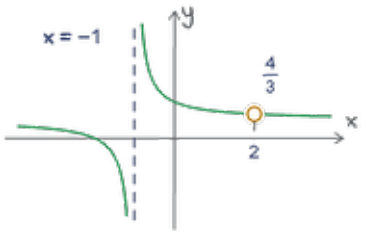
At -1 → top $x+2\to 1$, bottom $x+1\to 0$

Think: $x\to -1^{\pm}\to \frac{1}{0^{\pm}}\to \pm\infty$

→ infinite, asymptote $x=-1$

IDEA 2: THE GRAPH, AND A REPEATED FACTOR

Key ideas



Rule: factor, cancel → bottom at a

Bottom ≠ 0 at a → removable

Bottom 0, top ≠ 0 → infinite

Repeated: $\frac{x-2}{(x-2)^2} = \frac{1}{x-2}$

Still 0 at 2, top 1

→ infinite, not a hole

PROBLEM 1 OF 2

Factor first

Let $f(x) = \dfrac{x^2 - 3x}{x^3 - 6x^2 + 9x}$.

(a) Name the type of discontinuity at $x = 0$.

(b) Name the type of discontinuity at $x = 3$.

Solution Factor, then the bottom

Plan: factor → what's left below

(a)

$$\lim_{x \to 0} (x^2 - 3x) = 0 - 0 = 0$$

$$\lim_{x \to 0} (x^3 - 6x^2 + 9x) = 0 - 0 + 0 = 0$$

Both → 0 → not done → factor

$$x^2 - 3x = x(x-3)$$

$$x(x^2 - 6x + 9) = x(x-3)^2$$

$x \to 0$: $x \neq 0$ and $x - 3 \neq 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\cancel{x}(x-3)}{\cancel{x}(x-3)(x-3)}$$

$$= \lim_{x \to 0} \frac{1}{x-3}$$

Bottom → $-3 \neq 0$ → plug in

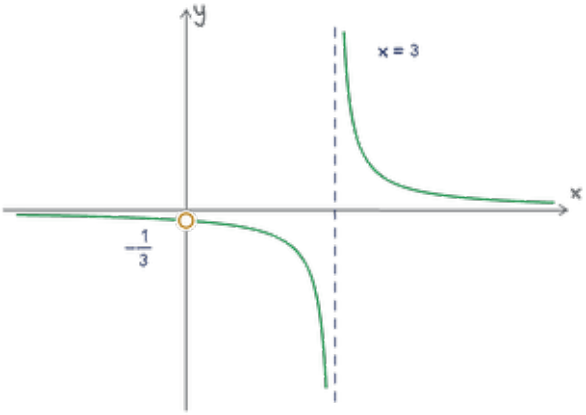
$$= \frac{1}{0-3} = -\frac{1}{3}$$

$f(0)$? undefined (bottom = 0)

Limit $-\frac{1}{3}$, $f(0)$ undefined → removable

Workbook → factor → holes

Solution Factor, then the bottom



(b)

Plan: reduced bottom at 3

$x \to 3$ → bottom $x - 3 \to 0$ (one stayed)

top $1 \neq 0$

Think: $x \to 3^{\pm} \to \frac{1}{0^{\pm}} \to \pm\infty$

Unbounded → infinite, asymptote $x = 3$

Graph → hole at 0, height $-\frac{1}{3}$

→ vertical asymptote at $x = 3$

Workbook → what's left below

Careful: $\frac{x(x-3)}{x(x-3)^2}$ → hole at 3?

✗ Common mistake

$f(x) = \frac{x(x-3)}{x(x-3)^2}$

$x-3$ cancels

→ a hole at 3

✓ Do this

$(x-3)^2$ → one stays

$\frac{1}{x-3}$

→ vertical asymptote

PROBLEM 2 OF 2

Read the breaks from a graph

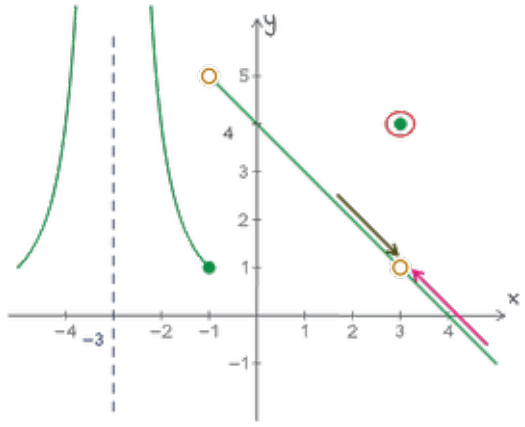
For which value of c does $\lim_{x\to c} f(x)$ exist, but f has a discontinuity at c?

(A) -3

(B) -1

(C) 1

(D) 3



Solution Both sides at each break

Plan: each break → both sides → same real number?

Limit exists + a break → removable

At 3 → both sides → 1

$f(3) = 4 \neq 1$ → removable

(A) -3 → both sides → ∞ → no real limit ✗

(B) -1 → left 1, right 5 → no limit ✗

(C) 1 → no break ✗

So (D) → $c = 3$

Workbook → name each break

WRAP-UP

Name a break? Sides first

Sides meet, $f(a)$ missing or off → removable

Both real, different → jump

A side → $\pm\infty$ → infinite

Rational → factor first

Repeated factor → one stays → not a hole

AP: limit exists, but a break → removable

Next → 1.11: continuity at a point

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.