# 1.10 / Types of Discontinuities

Watch the lesson ➤

*Exploring types of discontinuities*

**Learning goal**
Name the type of a discontinuity (removable, jump, or infinite) from a graph, a table, or a formula, and give the reason with limits.

## Key ideas

01 Name a break from the sides first: do both sides head to the same real number? On the graph, at $-1$ yes, $\frac{3}{2}$, but $f(-1) = 3$: **removable**. At 1, $\frac{5}{2} \neq 1$, both real: **jump**. At 3 a side heads to $\pm\infty$: **infinite**.

02 AP: "$\lim_{x \to -1} f(x) = \frac{3}{2} \neq f(-1)$, so removable"; "$\lim_{x \to 3^-} f(x) = -\infty$, an infinite discontinuity due to a vertical asymptote."

03 A zero in the bottom, hole or asymptote: $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2} = \dfrac{(x-2)(x+2)}{(x-2)(x+1)}$; the bottom is 0 at $x = 2$ and $x = -1$. $x \to 2$ means $x \neq 2$, so $f(x) = \dfrac{x+2}{x+1}$ and $\lim_{x \to 2} f(x) = \frac{4}{3}$, while $f(2)$ is undefined: **removable**. At $-1$ the top $x + 2 \to 1$, the bottom $x + 1 \to 0$. Think: $\frac{1}{0} \to \pm\infty$: **infinite**.

04 In general: **factor** and cancel. The reduced bottom $\neq 0$ at $a$: removable. The reduced bottom 0, top $\neq 0$: infinite. A repeated factor, $\dfrac{x-2}{(x-2)^2} = \dfrac{1}{x-2}$, is still 0 at 2: infinite, not a hole.

### Remember

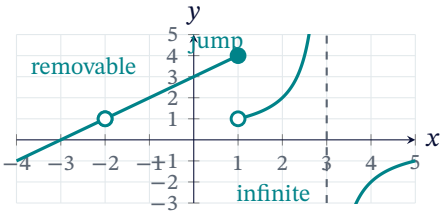
**removable discontinuity**
The limit exists, but the value is missing or somewhere else: a hole.

**jump discontinuity**
The left-hand and right-hand limits are different real numbers.

**infinite discontinuity**
At least one side grows without bound: a vertical asymptote.

**vertical asymptote**
A vertical line $x = a$ that the graph follows up or down without bound.

## The three types




Removable at $x = -2$ (both sides head to 1, no dot), jump at $x = 1$ (the sides head to 4 and 1), infinite at $x = 3$ (a vertical asymptote).

Class notes

# 1.10 / Write it clearly

*What earns the point on the AP Exam, and the mistakes that lose it.*

## Writing checklist

Notation to use and to avoid

| WRITE | AVOID |
|---|---|
| Name the type with limits: "$\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = 5$ are different real numbers, so $f$ has a jump discontinuity at $x = 1$." A shape word such as "gap" is not a reason. | Do not write a limit equal to $\frac{0}{0}$. Write "direct substitution gives the 0/0 form," then factor. |
| Keep lim on every line until you substitute: $\lim_{x\to 3} \frac{(x+2)(x-3)}{x-3} = \lim_{x\to 3}(x+2) = 5$. | Do not do arithmetic with $\infty$, as in "$\frac{5}{0} = \infty$." Say that $f(x)$ increases or decreases without bound. |

## Common mistakes

The mistake code is used in the answers and in My Mistakes

### 01 A zero bottom is not always an asymptote

Code L9

✗ $f(x) = \dfrac{x^2 - x - 20}{x - 5}$ has a vertical asymptote at $x = 5$, because the bottom is 0 there.

✓ $x^2 - x - 20 = (x - 5)(x + 4)$, so $x - 5$ cancels. The limit is 9, a real number, and $f(5)$ is undefined: a hole, removable.

### 02 A moved dot is not a jump

Code L2

✗ $\lim_{x\to 2^-} f(x) = 4$, $\lim_{x\to 2^+} f(x) = 4$, and $f(2) = 1$, so $f$ has a jump at $x = 2$.

✓ A jump needs two different one-sided limits. Both are 4, so the limit is 4; only the value is elsewhere: removable.

### 03 One unbounded side makes it infinite

Code L8

✗ $\lim_{x\to 3^-} f(x) = -1$ and $f(x)$ increases without bound as $x \to 3^+$. The sides are different, so this is a jump.

✓ A jump needs two real numbers, and $\infty$ is not a real number. One unbounded side is enough: an infinite discontinuity.

**Real AP questions on this topic** Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 tests the types of discontinuities. The 2012 Course Description multiple-choice question AB 17 reads a graph and asks where a limit exists but the function is not continuous.

# 1.10 / Guided examples

*Watch, pause, and write the reasoning. Full solutions are in the video.*

## Problem types

Read the problem, name the type, then use its method

| Type | Method | Problems |
|---|---|---|
| Name the type from a graph | Follow each side to $x = a$. Same height but the dot is missing or elsewhere: removable. Two different heights: jump. A side grows without bound: infinite. | Practice 1 |
| The limit exists, but there is a break | Both sides head to the same real number $L$, but $f(a)$ is undefined or $f(a) \neq L$: removable. | E02 |
| Rational functions: factor first | Factor and cancel. $x - a$ cancels from the bottom completely: hole. $x - a$ still in the bottom: vertical asymptote. | E01 · Practice 2 |
| A boundary of a piecewise function | Each one-sided limit with its own piece. Different real numbers: jump. The same $L$ but $f(a) \neq L$ or undefined: removable. | Practice 4 |
| What a table suggests | Read both sides. The same number $L$ from both sides: compare it with $f(a)$; undefined or different means removable. Two different numbers: a jump. Values that grow without bound: infinite. A table only suggests. | Practice 5 |
| Match a type to limit facts or to a formula | Turn each fact into a picture: same real number $L$, two different real numbers, or unbounded. | Practice 3 |

Example 01

Video · E01

Let $f(x) = \dfrac{x^2 - 3x}{x^3 - 6x^2 + 9x}$.
(a) Name the type of discontinuity at $x = 0$.
(b) Name the type of discontinuity at $x = 3$.

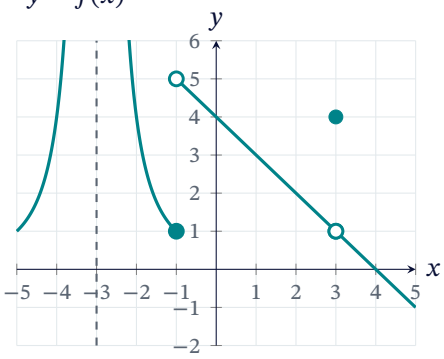
Plan: factor, then the bottom

## Example 02

Video · E02

For which $c$ does $\lim_{x \to c} f(x)$ exist, but $f$ has a discontinuity at $c$?

$y = f(x)$




A. $-3$
B. $-1$
C. $1$
D. $3$

> Check both sides at every break.

**VIDEO COMPLETE** Check your example answers at the end of this booklet.

# 1.10 / Example answers

*Types of Discontinuities*

E01 (a) removable; (b) infinite

E02 (D)