1.8 / Practice workbook

The Squeeze Theorem

Practice

Problems 1–6 / Show your reasoning

1 For all $x \neq 1$ near 1,
$3 - |x-1| \leq f(x) \leq 3 + (x-1)^2$, and $f(1) = 4$.
Which of the following must be true?
I. $\lim_{x \to 1} f(x) = 3$
II. $\lim_{x \to 1} f(x) = f(1)$
III. $\lim_{x \to 1} \dfrac{f(x)}{x+2} = 1$

A. I only
B. II only
C. I and III only
D. II and III only

1.8 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} k + x^2 \cos\dfrac{3}{x}, & x < 0 \\ \dfrac{\sin 12x}{4x}, & x > 0 \end{cases}$, where k is a constant.

(a) Use the squeeze theorem to find $\lim_{x \to 0^-} f(x)$.

(b) Find $\lim_{x \to 0^+} f(x)$.

(c) For what value of k does $\lim_{x \to 0} f(x)$ exist? Explain.

3

What is $\lim_{x \to 1} \dfrac{\sin(x-1)}{x^2+2x-3}$?

A. $\dfrac{1}{4}$

B. 1

C. 0

D. nonexistent

1.8 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

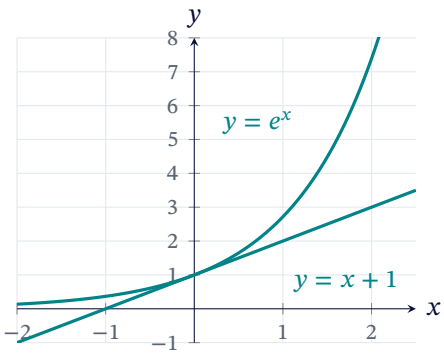
Let $f(x) = \dfrac{1 - \cos x}{x^2}$ for $0 < |x| < \pi$.

(a) Show that $f(x) = \dfrac{\sin x}{x} \cdot \dfrac{\sin x}{x} \cdot \dfrac{1}{1 + \cos x}$.

(b) Find $\lim_{x \to 0} f(x)$.

(c) Use (b) to find $\lim_{x \to 0} \dfrac{1 - \cos 6x}{x^2}$.

5 The graphs of $y = x + 1$ and $y = e^x$ are shown. A function f satisfies $x + 1 \le f(x) \le e^x$ for all x. Which of the following must be true?
I. $\lim_{x \to 0} f(x) = 1$
II. $\lim_{x \to 2} f(x)$ exists.
III. $f(0) = 1$



A. I only
B. I and II only
C. I and III only
D. I, II, and III

6 Which of the following are true? I. $\lim_{x \to \infty} \frac{\sin 2x}{x} = 0$ II. $\lim_{x \to \frac{\pi}{4}} \frac{\sin 2x}{x} = 2$ III. $\lim_{x \to \infty} x \sin \frac{3}{x} = 3$

A. I only
B. II only
C. I and III only
D. I, II, and III

1.8 / Answer key

The Squeeze Theorem

1 (C)

2 (a) k: $k - x^2 \le f(x) \le k + x^2$ for $x < 0$, and both bounds approach k; (b) 3; (c) $k = 3$: the two one-sided limits must be the same real number

3 (A)

4 (a) $\dfrac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \dfrac{1-\cos^2 x}{x^2(1+\cos x)} = \dfrac{\sin^2 x}{x^2(1+\cos x)}$; (b) $\frac{1}{2}$; (c) 18

5 (C)

6 (C)