1.7-1.9 / Practice workbook

Choosing a Method

Practice

Problems 1–8 / Show your reasoning

1 Which of the following are true?

I. $\lim_{x\to 2} \dfrac{x^2-4}{x^2+4} = 0$

II. $\lim_{x\to 3} \dfrac{\sqrt{x+13}-4}{x-3} = \dfrac{1}{4}$

III. $\lim_{x\to \frac{\pi}{2}} \dfrac{\sin 2x}{\cos x} = 2$

A. I only
B. I and II only
C. I and III only
D. I, II, and III

2 What is $\lim_{x\to 0} \dfrac{\sin 2x}{\tan x}$?

A. $\dfrac{1}{2}$
B. 1
C. 2
D. nonexistent

1.7·1.9 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{\sqrt{6-x}-2}{x-2}$. Selected values of g are given in the table.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$g(x)$	2.1	2.01	2.001	5	1.999	1.99	1.9

(a) Estimate $\lim_{x\to 2} g(x)$.

(b) Using (a) for $\lim_{x\to 2} g(x)$, find $\lim_{x\to 2} \dfrac{g(x)}{f(x)}$.

1.7·1.9 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} \dfrac{x-1}{\sqrt{x+24}-5}, & x < 1 \\ 6, & x = 1 \\ \dfrac{2x^2+6x-8}{x-1}, & x > 1 \end{cases}$

(a) Find $\lim_{x \to 1^-} f(x)$.

(b) Find $\lim_{x \to 1^+} f(x)$.

(c) Does $\lim_{x \to 1} f(x)$ exist? Does it equal $f(1)$?

1.7·1.9 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{\sqrt{5 + 4\cos x} - 1}{1 + \cos x}$.

(a) Find $\lim_{x \to 0} f(x)$.

(b) Find $\lim_{x \to \pi} f(x)$. Show the work that leads to your answer.

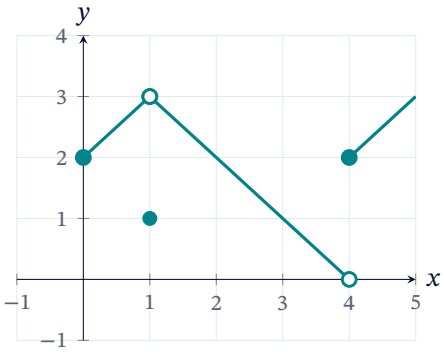
6 The graph of f is shown, and $g(x) = \dfrac{x-1}{\sqrt{x}-1}$.

Which of the following are true?

I. $f(x)g(x) \to 6$ as $x \to 1$.

II. As $x \to 4^-$, $f(x) + g(x)$ approaches 5.

III. $f(x)g(x)$ has no limit at $x = 4$.



A. I only

B. I and III only

C. II and III only

D. I, II, and III

1.7·1.9 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Find each limit, or explain why it does not exist.

(a) $\lim_{x \to 2} \dfrac{x+1}{x-2}$

(b) $\lim_{x \to 2} \dfrac{x^2 - 2x}{|x-2|}$

1.7·1.9 / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

Let $g(x) = \dfrac{\dfrac{1}{\sqrt{x+1}} - \dfrac{1}{2}}{x-3}$.

(a) Write $\frac{1}{\sqrt{x+1}} - \frac{1}{2}$ as a single fraction.

(b) Find $\lim_{x \to 3} g(x)$.

1.7·1.9 / Answer key

Choosing a Method

1 (C)

2 (C)

3 (a) 2; (b) -8

4 (a) 10; (b) 10; (c) Yes, $\lim_{x \to 1} f(x) = 10$, because both one-sided limits are 10; it is not equal to $f(1) = 6$

5 (a) 1; (b) 2

6 (B)

7 (a) does not exist: $-\infty$ from the left, ∞ from the right; (b) does not exist: -2 from the left, 2 from the right

8 (a) $\dfrac{2 - \sqrt{x+1}}{2\sqrt{x+1}}$; (b) $-\dfrac{1}{16}$