Lecture Notes · Unit 1 · Lecture 9 AP CED 1.7·1.9

Choosing a method

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INTRO: WHICH METHOD DO YOU PICK?

Choosing a Method AP Calculus 1.7 · 1.9

1.6 → tool given → exam: no tool

Plug in, factor, conjugate, identity, sides

Hard part → choosing

First line: plug in → a number? $\frac{c}{0}$? $\frac{0}{0}$? → picks the tool

No new ideas → 8 problems → name the method first

PROBLEM 1 OF 8

Plug in

Find $\lim_{x\to -1} \dfrac{x^2+4x}{x-2}$.

Solution Plug in

Plan: plug in

$-1-2=-3\neq 0$ (bottom)

$$\lim_{x\to -1} \frac{x^2+4x}{x-2} = \frac{1-4}{-3} = 1$$

Bottom $\neq 0$ → plug in, done (1.5)

PROBLEM 2 OF 8

Top to a number, bottom to 0

Find $\lim_{x\to -2} \dfrac{x-3}{x+2}$,
or explain why it does not exist.

Solution Top to a number, bottom to 0: each side

Plan: plug in → $\frac{c}{0}$

Top → $-2-3=-5$, bottom → $-2+2=0$

Think: $x\to -2^+ \to \frac{-5}{0^+} \to -\infty$

$x\to -2^- \to \frac{-5}{0^-} \to \infty$

Right $-\infty$, left ∞ → does not exist

$\frac{c}{0}$, $c\neq 0$ → each side, Think (1.3)

PROBLEM 3 OF 8

Zero over zero: factor

Find $\lim_{x\to -3} \dfrac{x^2+x-6}{x^2-9}$.

Solution Zero over zero with polynomials: factor

$9-3-6=0$, $9-9=0 \to \frac{0}{0}$

So, no limit?

Not yet → find $(x+3)$ → factor

$$\lim_{x\to -3} \frac{x^2+x-6}{x^2-9} = \lim_{x\to -3} \frac{(x+3)(x-2)}{(x+3)(x-3)}$$

$x\neq -3 \to x+3\neq 0$

$$= \lim_{x\to -3} \frac{x-2}{x-3} = \frac{-5}{-6} = \frac{5}{6}$$

$\frac{0}{0}$ → not "no limit"

X Common mistake

$\frac{0}{0}$

→ no limit

✓ Do this

$\frac{0}{0}$ → rewrite

Factor → $\frac{5}{6}$

Problem 2 → $\frac{c}{0}$ → each side · $\frac{0}{0}$ → rewrite

PROBLEM 4 OF 8

Zero over zero: the conjugate

Find $\lim_{x\to 5}\dfrac{\sqrt{x+4}-3}{x-5}$.

Solution Zero over zero with a root: the conjugate

$\sqrt{9}-3=0,\ 5-5=0 \to \frac{0}{0}$ → a root → conjugate

2 terms, flip sign → $C=\sqrt{x+4}+3$

bottom: keep $(x-5)$

$$\left(\sqrt{x+4}-3\right)\left(\sqrt{x+4}+3\right)=x+4-9=x-5$$

$x\neq 5 \to x-5\neq 0$

$$\lim_{x\to 5}\frac{x-5}{(x-5)C}$$

$$=\frac{1}{\sqrt{9}+3}=\frac{1}{3+3}=\frac{1}{6}$$

PROBLEM 5 OF 8

Zero over zero: a trig identity

Find $\lim_{x\to 0}\dfrac{\sin 2x}{\sin x}$.

Solution Zero over zero with sine: an identity

$\sin 0=0$ (top and bottom) → $\frac{0}{0}$ → $2x, x$ → identity

$\sin 2x=2\sin x\cos x$

Near 0, $x\neq 0 \to \sin x\neq 0$

$$\lim_{x\to 0}\frac{2\sin x\cos x}{\sin x}$$

$$=\lim 2\cos x=2\cos 0=2\cdot 1=2$$

$\frac{0}{0}$, sine → identity

PROBLEM 6 OF 8

Zero over zero: a fraction inside a fraction

Find $\lim_{x\to 2}\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

Solution

Fractions inside a fraction: common denominator, then cancel

$\frac{1}{2}-\frac{1}{2}=0,\ 2-2=0 \to \frac{0}{0}$

Fractions inside → common denominator, find $(x-2)$

$$\frac{1}{x}-\frac{1}{2}=\frac{2-x}{2x}$$

$2-x=-(x-2)$ → top = $\frac{-(x-2)}{2x}$

$\div(x-2)=\times\frac{1}{x-2}$

$$\lim_{x\to 2} \frac{-(x-2)}{2x} \cdot \frac{1}{x-2}$$

$x \neq 2 \to x-2 \neq 0$

$$= \lim_{x\to 2} \frac{-(x-2)}{2x(x-2)}$$

$$= \lim_{x\to 2} \frac{-1}{2x}$$

Bottom → $4 \neq 0$ → plug in

$$= \frac{-1}{2(2)} = -\frac{1}{4}$$

$\frac{0}{0}$, fractions inside → common denominator

PROBLEM 7 OF 8

Bars: each side

Find $\lim_{x\to -3} \frac{x^2+3x}{|x+3|}$,
or explain why it does not exist.

Solution Absolute value bars: each side

$\frac{0}{0}$, inside = 0 at −3 → factor, then each side

top = $x(x+3)$

$x+3 > 0$

$$\lim_{x\to -3^+} \frac{x(x+3)}{x+3} = -3$$

$x+3 < 0$

$$\lim_{x\to -3^-} \frac{x(x+3)}{-(x+3)} = -(-3) = 3$$

$-3 \neq 3$ → does not exist

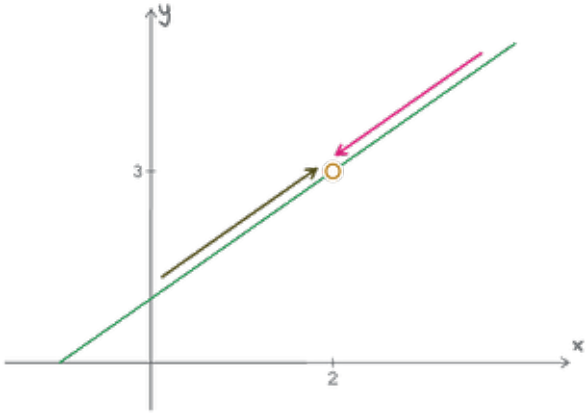
Bars → each side

PROBLEM 8 OF 8

A graph and a formula

The graph of f is shown.

Find $\lim_{x\to 2} \left[f(x) \cdot \frac{x^2-4}{x-2}\right]$.



Solution A graph and a formula: the graph's limit

Plan: each limit → product law

Graph: $x \to 2$ from the left, right → 3

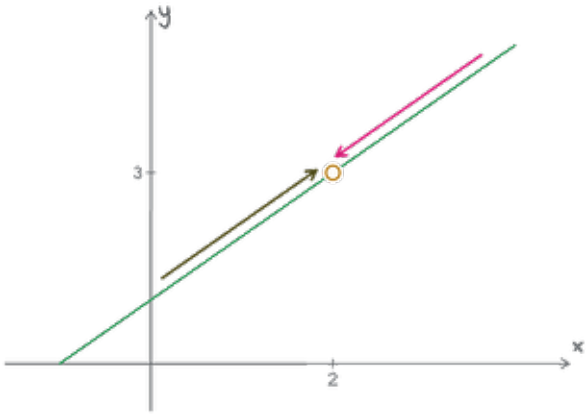
Hole → not the point

$$\lim_{x\to 2} f(x) = 3$$

Table → same job: both sides

Solution

A graph and a formula: the formula's limit, then the product law



$\frac{0}{0}$ at $x = 2$

$x^2 - 4 = (x-2)(x+2)$

$x \neq 2 \rightarrow x - 2 \neq 0$

$$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)}$$

$$= \lim_{x \to 2} (x+2) = 4$$

Both exist → product law

Limit = $3 \cdot 4 = 12$

2 pieces → each limit, then laws

WRAP-UP

Which tool?

Plug in first, then read what you get

1. One formula on both sides, a number → the limit
2. c over zero, c not zero → each side, Think
3. Zero over zero, polynomials → factor
4. Zero over zero, a root → conjugate
5. Zero over zero, sine → identity
6. Fractions inside, zero over zero → common denominator
7. Bars, or a boundary between two pieces → each side
8. Graph + formula → each limit, then the laws

Eight problems, one skill choosing

Next → 1.8 → the squeeze theorem

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.