1.7·1.9 / Choosing a Method

Watch the lesson ➤

Selecting procedures for determining limits; connecting multiple representations of limits

Learning goal
Look at any limit and know which tool to pick, from what plugging in gives; read limits from graphs and tables too.

Key ideas

01 Plug in first. You get one of three: a **number**, $\frac{c}{0}$ with $c \neq 0$, or $\frac{0}{0}$.

02 A number (the bottom is not 0): that is the limit. (Example 01)

03 $\frac{c}{0}$: one side at a time. Think: $\frac{c}{0^+}$, $\frac{c}{0^-}$. Opposite ways: no limit. (Example 02)

04 $\frac{0}{0}$: not "no limit". Find the hidden $(x - a)$: polynomials, **factor** (E03); a root, the conjugate (E04); sine and cosine, an identity (E05).

05 A fraction inside a fraction: combine the small fractions over a common denominator, then simplify. (Example 06)

06 A boundary or absolute value bars: each **side** on its own. (Example 07)

07 A graph or a table: read toward a from both sides; the value at a does not count. (Example 08)

Remember

direct substitution
Putting $x = a$ into a formula. For a polynomial, or a rational function whose bottom is not 0 at a, the result is the limit.

0/0 form
What direct substitution gives when the top and the bottom both approach 0. It is not a value: rewrite.

representation
A way to show a function or a limit: a graph, a table, a formula, or words.

estimate
A value read from a table. It suggests the limit; it does not prove it.

Method chooser

first: a graph, a table, or a formula? which kind of function?
polynomial, or rational with bottom $\neq 0$ at a : substitute
say what substitution gives before you rewrite
0/0 form with a polynomial: factor and cancel
0/0 form with $\sqrt{A} - b$: multiply by $\sqrt{A} + b$
0/0 form with sine or cosine: use an identity
boundary of a piecewise function: both one-sided limits

Class notes

1.7·1.9 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Keep lim on every line until you substitute. Write $\lim_{x\to a}\frac{(x-a)\,q(x)}{x-a} = \lim_{x\to a} q(x)$, not $= q(x)$.	Do not write a limit equal to $\frac{0}{0}$. The 0/0 form is not a value.
Put the side on x for a one-sided limit: $\lim_{x\to 3^-} f(x)$.	Do not use a table value at $x = a$, or the filled dot on a graph, as the limit.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The 0/0 form means rewrite

Code L1

✗ Direct substitution in $\frac{x^2-7x+12}{x-3}$ at $x = 3$ gives the 0/0 form, so the limit does not exist.

✓ For $x \neq 3$, $\frac{(x-3)(x-4)}{x-3} = x-4$, so the limit is $3-4=-1$.

02 Each side uses its own piece

Code L3

✗ $f(x) = x^2 - 1$ for $x < 2$ and $f(x) = x + 2$ for $x \geq 2$, so $\lim_{x\to 2} f(x) = f(2) = 4$.

✓ Left: $x^2 - 1 \to 3$. Right: $x + 2 \to 4$. The sides differ, so $\lim_{x\to 2} f(x)$ does not exist.

03 The filled dot is the value, not the limit

Code L2

✗ From both sides the curve heads to an open circle at $(-2, 3)$, and there is a filled dot at $(-2, 0)$. So $\lim_{x\to -2} f(x) = 0$.

✓ Both sides head to 3, so $\lim_{x\to -2} f(x) = 3$. The dot gives only the value $f(-2) = 0$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Topics 1.7 and 1.9 have no essential knowledge statements, so no released question is tied to them alone. Released multiple-choice questions read limits from graphs (CED sample Q11, CD-2012 AB Q17), which this lesson practices with formulas beside the graph.

1.7·1.9 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Substitute first	For a polynomial or rational formula, put in $x = a$; a nonzero bottom means the quotient law gives the limit.	E01, E02
The 0/0 form — choose the rewrite	Polynomial: factor. $\sqrt{A} - b$: conjugate. Sine and cosine: identity.	E03, E04, E05 · Practice 1, 2, 5
A boundary of a piecewise function	Each side uses its own piece; a 0/0 piece is rewritten first.	E07 · Practice 4, 7
Graph to limit statements	Follow the curve from each side; the filled dot is only the value.	
Table and formula	The table estimates; algebra on the formula gives the exact limit.	E08
Words and notation	Turn each phrase into a symbol: the side on x, the value after the equals sign.	
Two representations in one limit	Find each limit from its own representation, then use the limit laws.	Practice 3, 6
A fraction inside a fraction	Combine the small fractions over a common denominator. Use a conjugate as well if a radical prevents cancellation.	E06 · Practice 8

Example 01

Video · E01

Find $\lim_{x \to -1} \dfrac{x^2 + 4x}{x - 2}$.

> Plug in first.

Example 02

Video · E02

Find $\lim_{x \to -2} \dfrac{x - 3}{x + 2}$,
or explain why it does not exist.

> The top heads to a number, the bottom to 0. One side at a time.

Example 03

Video · E03

Find $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x^2 - 9}$.

Both are 0 at -3. Look for $(x + 3)$.

Example 04

Video · E04

Find $\lim_{x \to 5} \dfrac{\sqrt{x + 4} - 3}{x - 5}$.

A root on top: its conjugate, on the top and the bottom.

Example 05

Video · E05

Find $\lim_{x \to 0} \dfrac{\sin 2x}{\sin x}$.

Write $\sin 2x$ with $\sin x$ in it.

Example 06

Video · E06

Find $\lim_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

> Put the two fractions in the numerator over the same denominator before looking for a factor.

Example 07

Video · E07

Find $\lim_{x\to -3} \dfrac{x^2+3x}{|x+3|}$,
or explain why it does not exist.

> Factor the top, then split the bars where the inside is 0.

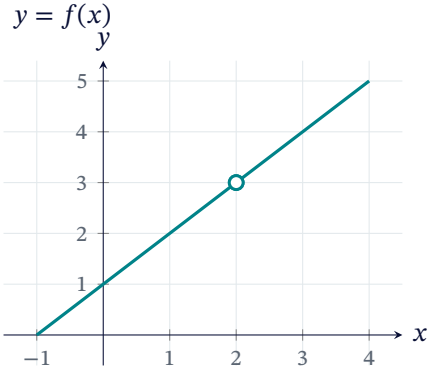
Example 08

Video · E08

The graph of f is shown.

Find $\lim_{x \to 2} \left[f(x) \cdot \frac{x^2 - 4}{x - 2} \right]$.

$y = f(x)$



> Find each limit first - one from the graph, one from the formula - then use the product law.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.7·1.9 / Example answers

Choosing a Method

E01 1

E02 does not exist: from the left ∞, from the right $-\infty$

E03 $\frac{5}{6}$

E04 $\frac{1}{6}$

E05 2

E06 $-\frac{1}{4}$

E07 does not exist: from the right -3, from the left 3

E08 12