1.5b / Practice workbook

One-Sided Limits of Piecewise Functions

Practice

Problems 1–7 / Show your reasoning

1

Let $f(x) = \begin{cases} x^2 - 3, & x < 2 \\ 5, & x = 2 \\ 3x - 5, & x > 2 \end{cases}$

Which of the following are true?

I. $\lim_{x \to 2} f(x) = 1$

II. $\lim_{x \to 2^+} f(x) = f(2)$

III. $\lim_{x \to 3} f(x) = 4$

A. I only

B. III only

C. I and III only

D. II and III only

1.5b / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $g(x) = \dfrac{(x+3)|x-1|}{x-1}$.

(a) Rewrite $g(x)$ without absolute value bars.

(b) Find $\lim_{x \to 1^-} g(x)$.

(c) Find $\lim_{x \to 1^+} g(x)$.

(d) Does $\lim_{x \to 1} g(x)$ exist? Explain.

3 Let c be a constant, and let

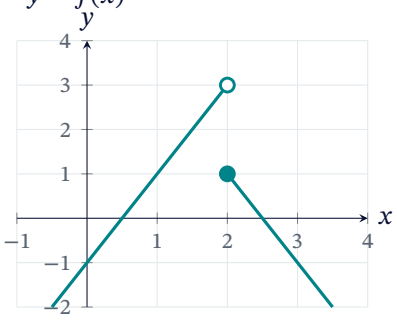
$$f(x) = \begin{cases} x^2, & x < c \\ 2x + 3, & x \geq c \end{cases}$$

For which values of c does $\lim_{x \to c} f(x)$ exist?

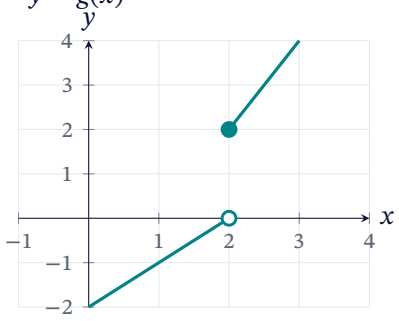
A. 3 only
B. -1 only
C. -1 and 3
D. no value of c

4 The graphs of f and g are shown. For which of the following does the limit as $x \to 2$ exist?

$y = f(x)$



$y = g(x)$



A. $f(x)$
B. $f(x) - g(x)$
C. $f(x) + g(x)$
D. $f(x)\,g(x)$

1.5b / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

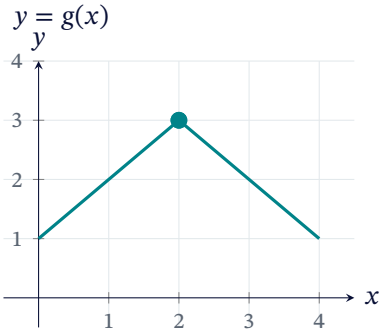
An oven is turned on at time $t = 0$, with t measured in minutes. Its temperature, in degrees Celsius, is $T(t) = \begin{cases} 20 + 40t, & 0 \le t < 5 \\ 220, & t \ge 5 \end{cases}$

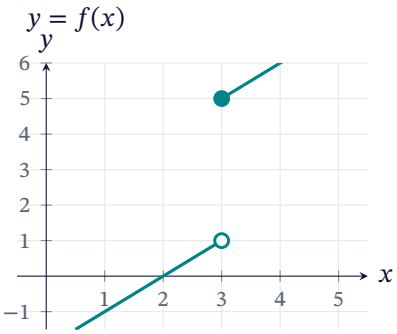
(a) Find $\lim_{t \to 5} T(t)$ using one-sided limits.

(b) Find the average rate of change of T on $[3, 7]$.

(c) Interpret your answer to (b) in context, with units.

6 The graphs of g (top) and f (bottom) are shown. What is $\lim_{x \to 2} f(g(x))$?





A. 1
B. 3
C. 5
D. nonexistent

1.5b / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The graph of f is shown.



(a) Find $\lim_{x \to -2^+} f(-x)$.

(b) Find $\lim_{x \to -2^-} f(-x)$.

(c) Does $\lim_{x \to -2} f(-x)$ exist? Explain.

1.5b / Answer key

One-Sided Limits of Piecewise Functions

1 (C)

2 (a) $g(x) = -(x+3)$ for $x < 1$, and $g(x) = x + 3$ for $x > 1$; (b) -4; (c) 4; (d) No: the one-sided limits -4 and 4 are not equal

3 (C)

4 (C)

5 (a) 220; (b) 20 degrees Celsius per minute; (c) From $t = 3$ to $t = 7$ minutes, the temperature of the oven increases by 20 degrees Celsius per minute on average

6 (A)

7 (a) As $x \to -2^+$, $-x \to 2^-$ (from the left), so $f(-x) \to 1$; (b) As $x \to -2^-$, $-x \to 2^+$ (from the right), so $f(-x) \to 3$; (c) No: the one-sided limits are 1 and 3, not equal