Lecture Notes · Unit 1 · Lecture 6 AP CED 1.5b

Piecewise one-sided limits

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INTRO: WHICH FORMULA AT A BOUNDARY?

Piecewise Limits AP Calculus 1.5

Two formulas, $x \to 2$ → which one?

The = in $x \geq 2$? Don't take the bait

Each side → right piece

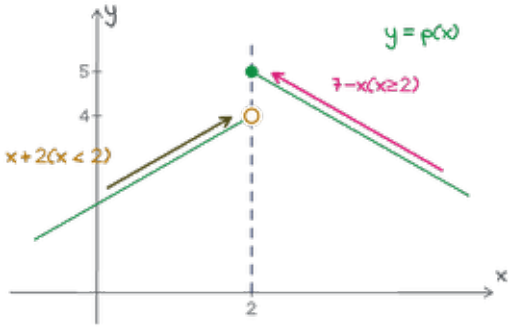
Even in $|\ |$, in $f(g(x))$

Let's get that 5

IDEA 1: EACH SIDE PICKS ITS OWN PIECE

At a boundary: which piece?

At a boundary → which one?



Each side → its own piece

$\lim_{x \to 2^-} p = 2 + 2 = 4$

$\lim_{x \to 2^+} p = 7 - 2 = 5$

$4 \neq 5$ → $\lim_{x \to 2} p(x)$ does not exist

IDEA 2: ABSOLUTE VALUE HIDES A BOUNDARY

Absolute value: a hidden boundary

Absolute value: where's the boundary?

$|\cdot|$ → boundary

e.g. $g(x) = \dfrac{|x-1|+|x+1|-2}{x-1}$

$x \to 1$ → split $|x-1|$ at 1 · $|x+1| = x+1$

$x \to 1^+$, $x - 1 > 0$ → $|x-1| = x-1$

$g = \dfrac{(x-1)+(x+1)-2}{x-1}$

$= \dfrac{2x-2}{x-1} = \dfrac{2(x-1)}{x-1} = 2$

$\lim_{x \to 1^+} g(x) = 2$

$x \to 1^-$, $x - 1 < 0$ → $|x-1| = -(x-1)$

$g = \dfrac{-(x-1)+(x+1)-2}{x-1}$

$= \dfrac{-x+1+x+1-2}{x-1} = 0$

$\lim_{x \to 1^-} g(x) = 0$

$0 \neq 2$

So $\lim_{x \to 1} g(x)$ does not exist

Split bars where inside = 0

On the AP → a graph, or a formula

IDEA 3: A FUNCTION INSIDE A FUNCTION

A function inside a function

A function inside a function

$f(g(x))$

$x \to 3$ → the inside first

$x \neq 3$ → heights 1.1, 1.01, ... → just above 1

$g(x) \to 1^+$ (from above)

1.01 on f's x-axis → where?
Right of 1 above → from the right
→ 1^+ → right piece, not the dot
Inside → where, from which side

PROBLEM 1 OF 4

Its own value at the boundary

Let $f(x) = \begin{cases} x^2 - 2x, & x < 3 \\ 6, & x = 3 \\ 2x - 3, & x > 3 \end{cases}$

Find (a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$

(c) $\lim_{x \to 3} f(x)$ (d) $f(3)$

Solution Each side its own piece

Plan: side → piece $f(3) = 6$ → (d)

(a)

$x \to 3^-, x < 3 \to x^2 - 2x$

$$\lim_{x \to 3^-} (x^2 - 2x)$$

$$= 3^2 - 2(3) = 9 - 6 = 3$$

(b)

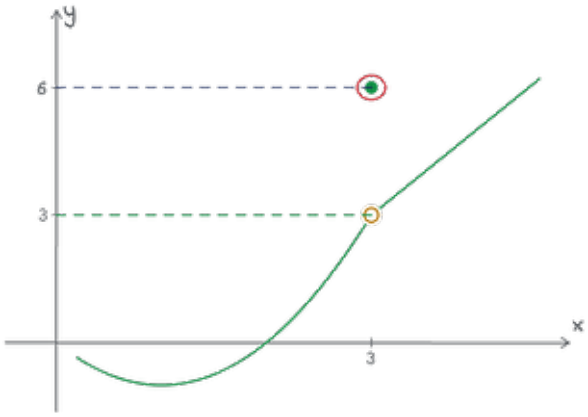
$x \to 3^+, x > 3 \to 2x - 3$

$$\lim_{x \to 3^+} (2x - 3)$$

$$= 2(3) - 3 = 6 - 3 = 3$$

Solution

Compare the sides; the value from the x = 3 line



Plan: sides → value

(c)

Both sides 3 → $\lim_{x \to 3} f(x) = 3$

(d)

$f(3)$ → line $x = 3$ → 6

Limit 3, value 6 → two questions

PROBLEM 2 OF 4

Split the bars

Let $g(x) = \dfrac{(x+1)\,|x-2|}{x-2}$.

Find (a) $\lim_{x \to 2^-} g(x)$ (b) $\lim_{x \to 2^+} g(x)$

(c) $\lim_{x \to 2} g(x)$, or explain why it does not exist.

Solution Split |x - 2| at 2 → the left side

Plan: inside = 0 at 2 → split → one side at a time

(a)

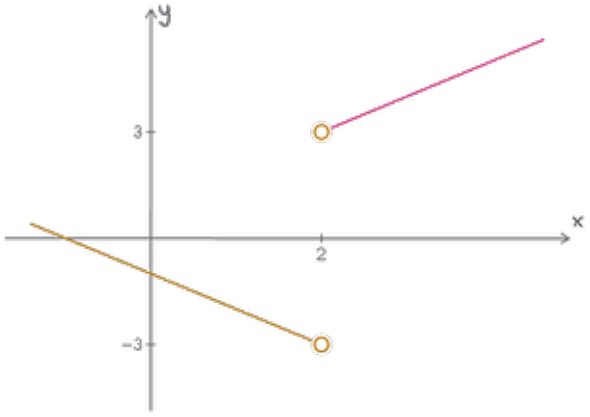
$x \to 2^-, x - 2 < 0 \to |x - 2| = -(x - 2)$

$$g(x) = \frac{(x+1) \cdot [-(x-2)]}{x-2}$$

$$\frac{x-2}{x-2} = 1 \quad \to \quad g(x) = -(x+1)$$

$$\lim_{x \to 2^-} g(x) = -(2+1) = -3$$

Solution Split |x - 2| at 2 → the right side, then compare



Plan: right → compare

(b)

$x \to 2^+, x-2>0 \to |x-2| = x-2$

$$g(x) = \frac{(x+1)(x-2)}{x-2}$$

$$= x+1, \text{ since } \frac{x-2}{x-2} = 1$$

$$\lim_{x \to 2^+} g(x) = 2+1 = 3$$

(c)

$-3 \neq 3$

So $\lim_{x \to 2} g(x)$ does not exist

Habit: split → cancel

Careful → where points are lost

X Common mistake

$g(x) = x+1$ (no bars)

→ 3 (right only)

✓ Do this

At 2 → two pieces

$$\lim_{x \to 2^-} g = -3 \neq 3 = \lim_{x \to 2^+} g$$

→ does not exist

PROBLEM 3 OF 4

Find k

Let $f(x) = kx - 1$ for $x < 2$ and $f(x) = x^2 + k$ for $x \geq 2$.

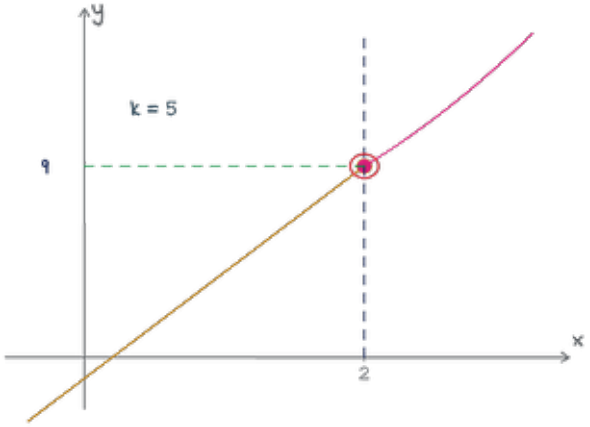
For what value of k does $\lim_{x \to 2} f(x)$ exist?

(A) 3

(B) 4

(C) 5

(D) no such value



Solution Set the two sides equal

Plan: exists ⇔ left = right

$$\lim_{x \to 2^-} f(x) = 2k - 1$$

$$\lim_{x \to 2^+} f(x) = 2^2 + k = 4 + k$$

$2k - 1 = 4 + k$

$k = 5 \quad 2(5) - 1 = 9 = 4 + 5$

So (C) → 5

If $1 + k = k + 3 \to 1 = 3 \to$ no k → (D)

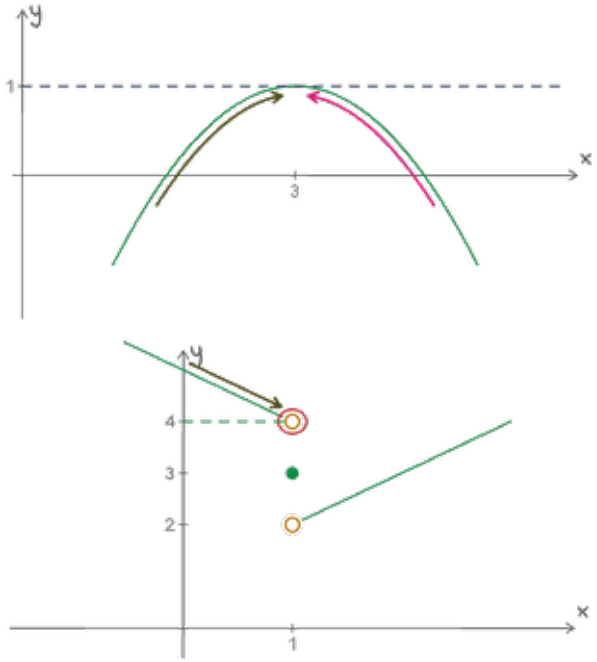
Habit: sides = first

PROBLEM 4 OF 4

A function inside a function

The graphs of g (top) and f (bottom) are shown. What is $\lim_{x\to 3} f(g(x))$?

(A) 1
(B) 2
(C) 4
(D) 3



Solution

Inside first: where it heads, and from which side

Plan: g first → f

x → 3 → g(x) → 1 from below

→ f's input → 1^-

$\lim_{x\to 3} f(g(x)) = 4$

(A) inside · (B) right · (D) f(1) ✗

So (C) → 4

WRAP-UP

At a boundary

Limit → each side, its own piece

Value → the piece with =

Limit → neighbors, value → the point

Bars → split where inside = 0, then cancel

Exam → each side's formula → compare

Inside a function → where, and from which side

Workbook · comments → Next: 1.6, bottom → 0

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.