1.5a / Practice workbook

Limit Laws

Practice

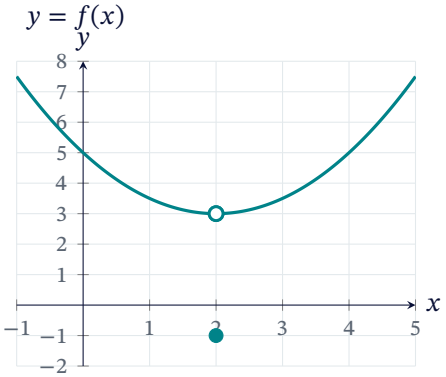
Problems 1–5 / Show your reasoning

1 If $\lim_{x\to -1} f(x) = 7$ and $\lim_{x\to -1} g(x) = -3$, what is $\lim_{x\to -1} \dfrac{[g(x)]^2 - f(x)}{\sqrt{f(x) - 3}}$?

A. -8
B. $\dfrac{1}{2}$
C. 1
D. nonexistent

2 If $\lim_{x\to 2} g(x) = 7$, what is $\lim_{x\to 2} \dfrac{g(x) + f(2)}{f(x)}$?

$y = f(x)$



A. -6
B. 2
C. $\dfrac{10}{3}$
D. nonexistent

3 Which of the following limits does not exist?

A. $\lim_{x\to 2} \dfrac{x^2 - 4}{x + 3}$
B. $\lim_{x\to -2} \dfrac{x^2 + 4}{x + 2}$
C. $\lim_{x\to 0} \dfrac{e^x}{x - 1}$
D. $\lim_{x\to 3} \sqrt{x^2 - 5}$

1.5a / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Suppose $\lim_{x \to 0} f(x) = 9$ and $\lim_{x \to 0} g(x) = -1$.

(a) Find $\lim_{x \to 0} e^{x\,g(x)} \sqrt{f(x) + 5g(x)}$.

(b) Can the root law be used to find $\lim_{x \to 0} \sqrt{g(x)}$? Explain.

1.5a / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Selected values of f are given in the table.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	3.8	3.98	3.998	1.999	1.99	1.9

(a) Does $\lim_{x \to 1} f(x)$ appear to exist?

(b) Estimate $\lim_{x \to 1}[(x-1)f(x)]$.

(c) Why can't the product law be used to find the limit in (b)?

1.5a / Answer key

Limit Laws

1 (C)

2 (B)

3 (B)

4 (a) 2; (b) No: the inside limit is -1, and the root law needs a positive inside limit

5 (a) No: the table suggests the left side heads to 4 and the right side heads to 2; (b) Using the table's one-sided estimates: left $0 \cdot 4 = 0$, right $0 \cdot 2 = 0$. So the table suggests $\lim_{x \to 1}[(x-1)f(x)] = 0$; (c) The product law needs both limits to exist as real numbers. The table cannot show that $\lim_{x \to 1} f(x)$ exists; it suggests the sides differ. So there is no basis for the product law