Lecture Notes · Unit 1 · Lecture 5 AP CED 1.5a

Limit laws

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: WHEN CAN WE SPLIT A LIMIT?

Limit Laws AP Calculus 1.5

1.4 table → suggests

1.5 for sure → limit laws

Too easy? → one check first

Split → when?

LAWS 1-4: CONSTANT, SUM, PRODUCT, QUOTIENT

The limit laws

As $x \to 3$, $f \to 4 = L$, $g \to -2 = M$ (real)

☆ Both limits exist (real) → then the law.

one side at a time, too.

1. Constant multiple

$$\lim_{x \to 3} cf = cL$$

$3f \to 3(4) = 12$

2. Sum and difference

$$\lim_{x \to 3} [f \pm g] = L \pm M$$

$f - g \to 4 - (-2) = 6$

3. Product

$$\lim_{x \to 3} fg = LM$$

$f \cdot g \to 4(-2) = -8$

4. Quotient

$$\lim_{x \to 3} \frac{f}{g} = \frac{L}{M}$$

if $M \neq 0$

$\frac{f}{g} \to g \to -2 \neq 0 \to \frac{4}{-2} = -2$

WHEN THE BOTTOM HEADS TO 0

When the bottom → 0

$f \to 4$, $\frac{f(x)}{x-3}$ ($x-3 \to 0$), $x \to 3$

$x \to 3^+$, $\frac{4}{0^+} \to \infty$

$x \to 3^-$, $\frac{4}{0^-} \to -\infty$

Top → not 0, bottom → 0 → unbounded → no real limit

$M = 0$ → no quotient law

Top → 0?

$\frac{0}{0}$ → not done yet

1.6

LAWS 5-7: POWERS, ROOTS, EXPONENTIALS

Powers, roots, exponentials

5. Powers

$$\lim_{x \to 3} [f(x)]^n = L^n$$

$n = 1, 2, 3, \ldots$

$[g(x)]^3$

$= g \cdot g \cdot g$

$(-2)(-2)(-2) = -8$ (product law)

6. Square roots

$$\lim_{x \to 3} \sqrt{f(x)} = \sqrt{L}$$

when $L > 0$

$\sqrt{f} \to \sqrt{4} = 2$

7. Exponentials

$$\lim_{x \to 3} e^{g(x)} = e^M$$

$e^{g(x)} \to e^{-2}$

5-7: limits of composite functions

Accept now → continuity later

Polynomial → plug in: $\lim_{x \to a} p(x) = p(a)$

(a quotient, too, if bottom ≠ 0)

PROBLEM 1 OF 2

A limit hidden in a sum

Suppose $\lim_{x \to 1} f(x) = 4$ and $\lim_{x \to 1} [2f(x) - g(x)] = 5$.

(a) Find $\lim_{x \to 1} g(x)$. Show your work.

(b) Is $\lim_{x \to 1} \dfrac{f(x)}{g(x) - 3}$ a real number? Explain.

Constant multiple

$\lim_{x \to 1} cf = cL$

Difference

$\lim_{x \to 1} [f - g] = L - M$

Quotient

$\lim_{x \to 1} \frac{f}{g} = \frac{L}{M}$ $(M \neq 0)$

Solution (a) Write g with pieces we know

(a)

Think: $2f \to 8 \to g \to 3$

Split $\lim_{x \to 1} [2f - g]$? needs $\lim_{x \to 1} g$ first → not yet

Plan: g from known pieces 2f, 2f − g

$g = 2f - [2f - g] = 2f - 2f + g = g$

$2f \to 2(4) = 8$ (const. multiple)

$2f - g \to 5$ (given)

Both limits exist, so the difference law applies.

$$\lim_{x \to 1} g = \lim_{x \to 1} 2f - \lim_{x \to 1} [2f - g]$$

$$= 8 - 5 = 3$$

Workbook → limits in sums

Solution (b) Bottom first

Constant multiple

$\lim_{x \to 1} cf = cL$

Difference

$\lim_{x \to 1} [f - g] = L - M$

Quotient

$\lim_{x \to 1} \frac{f}{g} = \frac{L}{M}$ $(M \neq 0)$

(b)

Plan: quotient → bottom first

$\lim_{x \to 1} [g(x) - 3] = 3 - 3 = 0$

Bottom → 0 → no quotient law

top → 4 → not $\frac{0}{0}$

Top → 4, bottom → 0 → $\frac{4}{0}$ → unbounded

(sign? not needed)

Not real → no

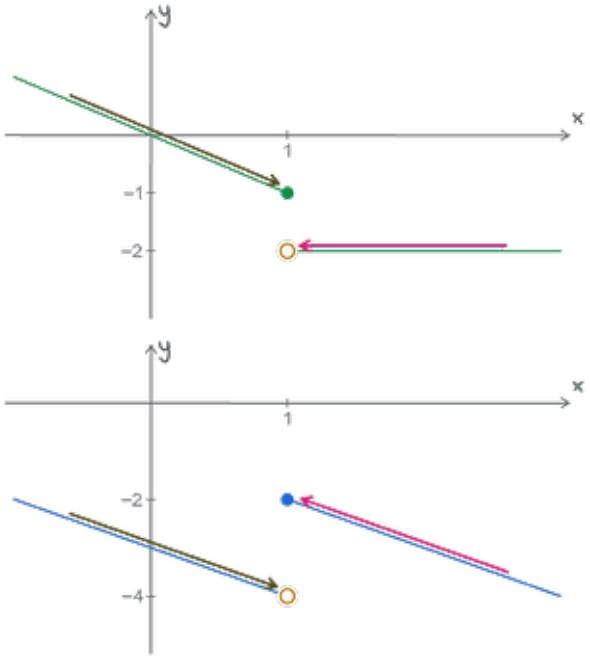
Workbook → bottom first

PROBLEM 2 OF 2

A product from two graphs

The graphs of f (top) and g (bottom) are shown. What is $\lim_{x\to 1} f(x)g(x)$?

(A) 8
(B) 4
(C) −4
(D) nonexistent



Solution One side at a time

Plan: both jump → one side at a time

→ Check each side

$x \to 1^- \to f \to -1, g \to -4$

$\lim_{x\to 1^-} fg = (-1)(-4) = 4$

$x \to 1^+ \to f \to -2, g \to -2$

$\lim_{x\to 1^+} fg = (-2)(-2) = 4$

Both sides 4 → $\lim_{x\to 1} f(x)g(x) = 4$

So (B) → 4

Workbook → one side at a time

WRAP-UP

When can we split a limit?

Each limit exists (real) → split

One side at a time, too

Quotient → bottom limit ≠ 0 first

Bottom → 0: no quotient law

Polynomial → plug in

Show the work: both limits exist, so the law applies

Conditions first → then the law

Next → 1.5, piecewise, one side at a time

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.