Lecture Notes · Unit 1 · Lecture 4 AP CED 1.4

Limits from tables

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: NO GRAPH, JUST A TABLE

Limits from Tables AP Calculus 1.4

No graph. Just numbers.

Read it right → a limit in seconds

Trust it too much → it lies

A table → a limit, for sure?

READING A TABLE NEAR X = 2

A table near x = 2

Limit: $f(x)$ as $x \to a$, $x \neq a$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	2.61	2.96	2.996	1	3.004	3.04	3.41

→ 3 ← 3

Left of 2 → toward 2

→ closing in on 3

One side? Not enough

Right of 2 → toward 2 → 3

Both sides → one number: 3

Middle column → $x \neq 2$ → skip it

That column → only $f(2) = 1$

Suggests: $\lim_{x \to 2} f(x) = 3$

RUNAWAY VALUES NEAR X = 4

Table near x = 4



Skip the middle column

Left → ×10 each step

→ no bound

$\lim_{x \to 4^-} f(x) = -\infty$

Right → toward 4, up

$\lim_{x \to 4^+} f(x) = \infty$

$-\infty \neq \infty$ → no limit at 4

A TABLE THAT LIES

A table near x = 0

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
f(x)	0	0	0	0	0	0

Every entry 0

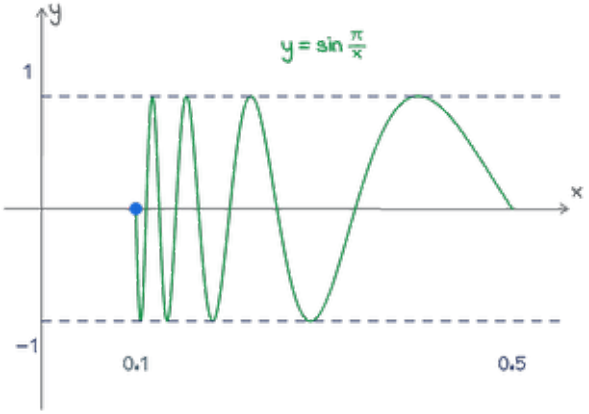
Guess → $\lim_{x \to 0} f(x) = 0$

Six zeros? $f(x) = \sin \frac{\pi}{x}$

x values → into $\frac{\pi}{x}$

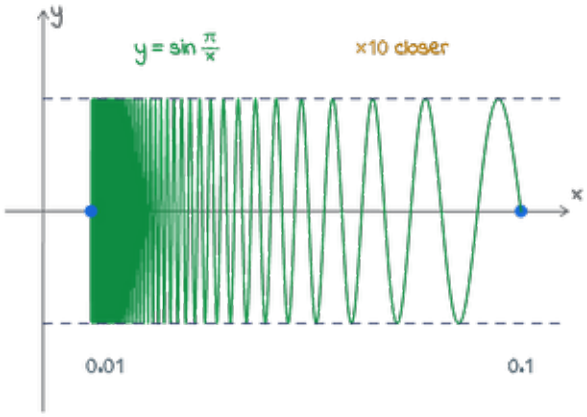
$\frac{\pi}{x} = \pm 10\pi, \pm 100\pi, \pm 1000\pi$

$\sin(\text{multiple of } \pi) = 0$ → all 0



$x = 0.1$ → height 0

Between: up to 1, down to −1



$x = 0.01$ → height 0

Zoom again → never settles

$\lim_{x \to 0} \sin \frac{\pi}{x}$ does not exist — not 0

The table missed it

only an estimate → 'suggests'

PROBLEM 1 OF 3

One side runs off

Selected values of f near $x = -2$ are given in the table.

(a) Describe $\lim_{x \to -2^-} f(x)$.

(b) Describe $\lim_{x \to -2^+} f(x)$.

(c) Describe $\lim_{x \to -2} f(x)$.

x	−2.1	−2.01	−2.001	−1.999	−1.99	−1.9
$f(x)$	−10	−100	−1000	3.001	3.01	3.1

→ −∞ | → 3

Solution (a) The left side

(a)

Plan: each side → a number, or no bound?

Left of −2 → $x < -2$

×10 each step → no bound

Suggests: $\lim_{x \to -2^-} f(x) = -\infty$

Solution (b) The right side

x	−2.1	−2.01	−2.001	−1.999	−1.99	−1.9
$f(x)$	−10	−100	−1000	3.001	3.01	3.1

→ 3

(b)

Right side: $x > -2$

Suggests: $\lim_{x \to -2^+} f(x) = 3$

Solution (c) Both sides

x	−2.1	−2.01	−2.001	−1.999	−1.99	−1.9
$f(x)$	−10	−100	−1000	3.001	3.01	3.1

→ −∞ ≠ → 3

(c)

No bound ≠ 3 → sides differ

$\lim_{x \to -2} f(x)$ does not exist

Workbook: read each side

PROBLEM 2 OF 3

One side only

Selected values of p for $x > 4$ are given in the table.

(a) Estimate $\lim_{x \to 4^+} p(x)$.

(b) A student writes $\lim_{x \to 4} p(x) = 9$. Does the table support this? Explain.

x	4.001	4.01	4.1
$p(x)$	9.004	9.04005	9.405

→ 9

Solution (a) Which side?

(a)

Plan: which side of 4? → then read it

Every $x > 4$ → right side only

Toward 4 → closing in on 9

Suggests: $\lim_{x \to 4^+} p(x) = 9$

Solution (b) The student's claim

x	4.001	4.01	4.1
p(x)	9.004	9.04005	9.405

→ 9

(b)
No $x < 4$ in the table

other side → unknown

→ not supported

workbook → which side?

PROBLEM 3 OF 3

What the table supports

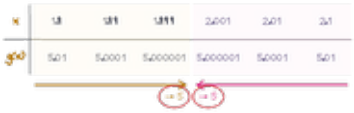
Which of the following does the table support?

(A) $g(2) = 5$

(B) $\lim_{x \to 2} g(x) = 5$

(C) $\lim_{x \to 2^-} g(x) = 5.01$

(D) $\lim_{x \to 2} g(x)$ does not exist



Solution One statement at a time

Plan: both sides → each choice

Left → heading to 5

Right → the same 5

(A) $g(2) = 5$? No $x = 2$ → no

(B) Both sides → 5 → supported

(C) 5.01: just one entry → no

(D) The sides agree → no

So (B)

workbook → what it supports

WRAP-UP

A limit, for sure?

No → a few snapshots, not the movie

Both sides; column $x = a$ → only $f(a)$

AP: a conjecture → 'the table suggests'

Next → 1.5: for sure, with algebra

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.