1.4 / Limits from Tables

Watch the lesson ➤

Estimating limit values from tables

Learning goal
Estimate one-sided and two-sided limits from a table, and tell what a table can and cannot show.

Key ideas

01 A table near $x = 2$: the entries with $x < 2$ close in on 2. Read toward 2: the outputs head to 3. The table is **evidence** for the limit.

02 Is one side enough? A limit is one real number from both sides. Read the entries with $x > 2$ toward 2 as well: read **both** sides.

03 The entry at $x = 2$: $x \to 2$ means $x \neq 2$, so skip it. That entry gives only the **value** $f(2) = 1$. The table suggests $\lim_{x\to 2} f(x) = 3$.

04 Near $x = 4$, skip the entry at 4. From the left the size grows ten times each step closer, past any size: no bound, so the table suggests $\lim_{x\to 4^-} f(x) = -\infty$. From the right, the same sizes, positive: $\lim_{x\to 4^+} f(x) = \infty$. The sides **differ**: no limit at 4.

05 Near 0 every entry of $f(x) = \sin\dfrac{\pi}{x}$ is 0: at $x = \pm 0.1, \pm 0.01, \pm 0.001$, $\dfrac{\pi}{x}$ is a multiple of π. But the graph from 0.5 to 0.1 goes up to 1 and down to -1 in between, and ten times closer it does the same: it never settles, so $\lim_{x\to 0} \sin\dfrac{\pi}{x}$ does not exist. The table **missed** it. A table is only an **estimate**: write "The table suggests".

Remember

conjecture
A guess backed by evidence. From a table, write: "The table suggests ...".

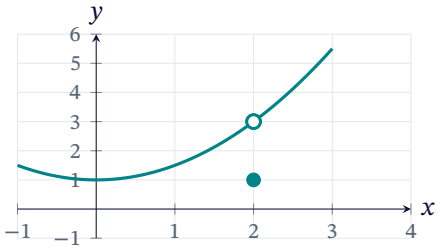
estimate
A careful guess from evidence, such as the values in a table.

left-hand limit
The number f(x) approaches as x comes toward a from values less than a.

right-hand limit
The number f(x) approaches as x comes toward a from values greater than a.

unbounded
The outputs keep growing (or keep falling) and never settle near one number.

A table near a point



Columns near $x = 2$ follow the curve to height 3, so the limit is about 3. The column at $x = 2$ is the dot: $f(2) = 1$.

Class notes

1.4 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write each side with its sign: $\lim_{x\to 2^-} f(x) = 4$ and $\lim_{x\to 2^+} f(x) = 4.5$.	Do not write a table entry as the limit, such as $\lim_{x\to 3} f(x) = 5.994$. The limit is the number the outputs approach.
When the limit comes from a table, say so: "The table suggests $\lim_{x\to 3} f(x) = 6$."	Do not write $\lim_{x\to 3} f(x) = f(3)$ just because the table has a column at $x = 3$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The column at x = a is not the limit

Code L2

✗ $\lim_{x\to 5} g(x) = 0$ because the column at $x = 5$ shows $g(5) = 0$.

✓ Read the columns near 5. If the outputs head to 8 from both sides, then $\lim_{x\to 5} g(x) = 8$; $g(5) = 0$ is only the value.

02 Check both sides

Code L3

✗ $\lim_{x\to -1} h(x) = 7$ because $h(-1.1)$, $h(-1.01)$, $h(-1.001)$ head to 7.

✓ Those columns give only $\lim_{x\to -1^-} h(x) = 7$. Read $x = -0.999, -0.99, -0.9$ too. If they head to another number, the limit does not exist.

03 All zeros can fool you

Code L8

✗ Every entry near 0 is 0, so $\lim_{x\to 0} f(x) = 0$.

✓ A table is only an estimate. $\sin\frac{\pi}{x}$ is 0 at $x = \pm 0.1, \pm 0.01, \ldots$ but swings to 1 and -1 in between: the limit does not exist.

04 Large outputs with opposite signs

Code L12

✗ The outputs near $x = 4$ are -500, -5000 on the left and 500, 5000 on the right, so $\lim_{x\to 4} f(x) = \infty$.

✓ Each side: $\lim_{x\to 4^-} f(x) = -\infty$ and $\lim_{x\to 4^+} f(x) = \infty$. The sides go opposite ways, so the limit does not exist.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ asks only for a limit from a table. The CED names "constructing a table of values to conjecture a limit" as a calculator use; in this book the values are given, so no calculator is needed.

1.4 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Example 01

Video · E01

Values of f near -2 are given.
(a) Describe $\lim_{x\to -2^-} f(x)$.
(b) Describe $\lim_{x\to -2^+} f(x)$.
(c) Describe $\lim_{x\to -2} f(x)$.

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
$f(x)$	-10	-100	-1000	3.001	3.01	3.1

> Each side on its own. Settling, or unbounded?

Example 02

Video · E02

Values of p for $x > 4$ are given in the table.
(a) Estimate $\lim_{x\to 4^+} p(x)$.
(b) A student writes $\lim_{x\to 4} p(x) = 9$. Supported?

x	4.001	4.01	4.1
$p(x)$	9.004	9.04005	9.405

> Which side does the table show?

Example 03

Video · E03

Which does the table support?

x	1.9	1.99	1.999	2.001	2.01	2.1
$g(x)$	5.01	5.0001	5.000001	5.000001	5.0001	5.01

A. $g(2) = 5$
B. $\lim_{x \to 2} g(x) = 5$
C. $\lim_{x \to 2^-} g(x) = 5.01$
D. $\lim_{x \to 2} g(x)$ does not exist

Plan: both sides, then each choice

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.4 / Example answers

Limits from Tables

E01 (a) The table suggests $\lim_{x \to -2^-} f(x) = -\infty$; (b) The table suggests $\lim_{x \to -2^+} f(x) = 3$; (c) does not exist: one side is unbounded, the other heads to 3

E02 (a) The table suggests $\lim_{x \to 4^+} p(x) = 9$; (b) No: the table shows only $x > 4$, so it says nothing about the left-hand limit; the two-sided limit needs both sides

E03 (B)