# 1.3 / Practice workbook

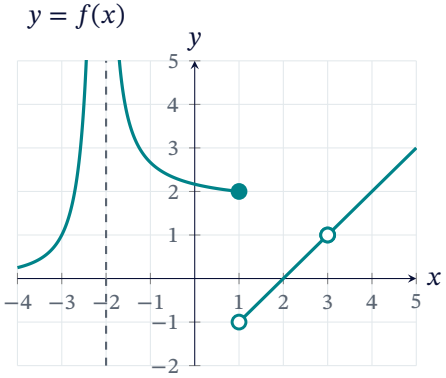
*Limits from Graphs*

## Practice

Problems 1–7 / Show your reasoning

1 The graph of $f$ is shown. Which of the following statements are true?
I. $\lim_{x \to -2} f(x) = \infty$
II. $\lim_{x \to 1^-} f(x) = f(1)$
III. $\lim_{x \to 3} f(x)$ does not exist.

$y = f(x)$




A. I only
B. I and II only
C. II and III only
D. I, II, and III

## 1.3 / AP-style free response

*Problem 2* ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = -\dfrac{2}{x}$.

**(a)** Find $\lim_{x \to 0^+} f(x)$.

**(b)** Find $\lim_{x \to 0^-} f(x)$.

**(c)** Does $\lim_{x \to 0} f(x)$ exist? Explain.

**(d)** Find $\lim_{x \to \infty} f(x)$.

## 1.3 / AP-style free response

*Problem 3* ➤ */ Show the work and the reasons a grader looks for.*

For $x > 0$, let $f(x) = \cos\left(\frac{1}{x}\right)$.

**(a)** Find $f\left(\frac{1}{2\pi}\right)$.

**(b)** Find $f\left(\frac{1}{3\pi}\right)$.

**(c)** Does $\lim_{x \to 0^+} f(x)$ exist? Explain.

## 1.3 / AP-style free response

*Problem 4* ➤ */ Show the work and the reasons a grader looks for.*

On $[-5, 5]$, a calculator graph of $f$ looks like the line $y = 2x + 1$. In fact, $f(x) = 2x + 1$ for $x \neq 1$, and $f(1) = 4$.

**(a)** Find $\lim_{x \to 1} f(x)$.

**(b)** Find $f(1)$.

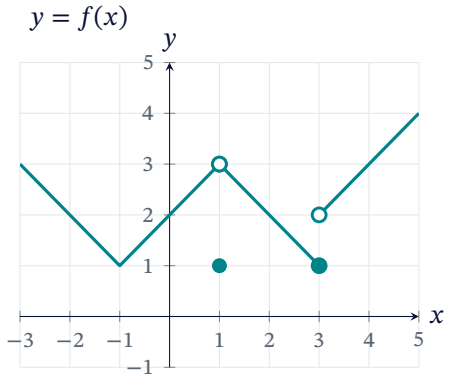
**(c)** Why does the calculator graph not show this?

## 1.3 / AP-style free response

*Problem 5* ➤ */ Show the work and the reasons a grader looks for.*

Sketch the graph of one function $f$ with all of the following:
$\lim_{x \to 1^-} f(x) = 2$, $\lim_{x \to 1^+} f(x) = -1$, $f(1) = -1$, and $\lim_{x \to 3} f(x) = \infty$.

6 The graph of $f$ is shown. Which of the following statements is true?




A. $\lim_{x \to -1} f(x)$ does not exist
B. $\lim_{x \to 1} f(x) = 3$
C. $\lim_{x \to 1} f(x) = f(1)$
D. $\lim_{x \to 3} f(x) = 2$

7 Suppose $\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to 3^+} f(x) = 2$, and $f(3) = 5$. Which of the following must be true?
I. $\lim_{x \to 3} f(x) = 2$
II. $\lim_{x \to 3} f(x) = f(3)$
III. $f(x)$ can be made as close to 2 as we like by taking $x$ close enough to 3, with $x \neq 3$.

A. I only
B. II only
C. I and III only
D. I, II, and III

# 1.3 / Answer key

*Limits from Graphs*

**1** (B)

**2** (a) $-\infty$; (b) $\infty$; (c) No: the two sides go opposite ways, so there is no limit, not even $\infty$ or $-\infty$; (d) 0

**3** (a) 1; (b) $-1$; (c) No: as $x \to 0^+$, $f(x)$ keeps swinging between 1 and $-1$ and never settles

**4** (a) 3; (b) 4; (c) At this scale one point (and one missing point) is too small to see; read what is given

**5** One example: $f(x) = x + 1$ for $x < 1$; $f(x) = \dfrac{1}{(x-3)^2} - \dfrac{5}{4}$ for $1 \le x < 3$; $f(x) = \dfrac{1}{(x-3)^2}$ for $x > 3$

**6** (B)

**7** (C)