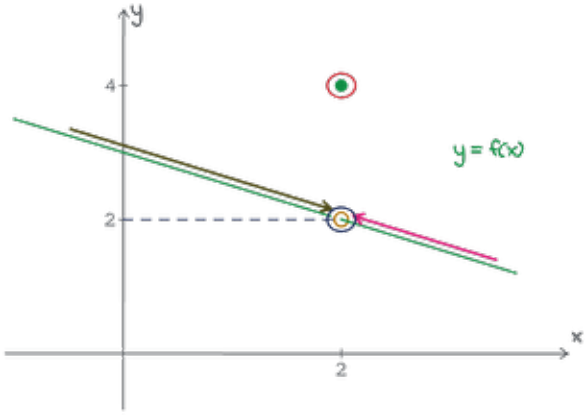
Lecture Notes · Unit 1 · Lecture 3 AP CED 1.3

Limits from graphs

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: WHEN IS THERE NO LIMIT?

Limits from Graphs AP Calculus 1.3



The dot at 4 → bait

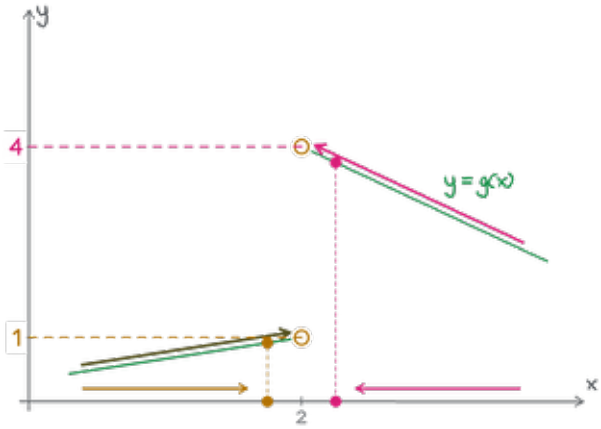
$\lim_{x \to 2} f(x) = 2$

Dot → $f(2) = 4$

Each side → 3 ways
a limit can fail

PICTURE 1: THE SIDES DISAGREE (ONE-SIDED LIMITS)

Left and right



Picture 1. Sides disagree

$y = g(x)$

From the left → 1

From the right → 4

$\lim_{x \to 2^-} g(x) = 1$

Small – up top → from the left, not negative

$\lim_{x \to 2^+} g(x) = 4$

→ one-sided limits

Limit: both sides → same real number

Left M, right N → M = N required

$\lim_{x \to 2^-} g(x) = 1 \neq 4 = \lim_{x \to 2^+} g(x)$

So $\lim_{x \to 2} g(x)$ does not exist

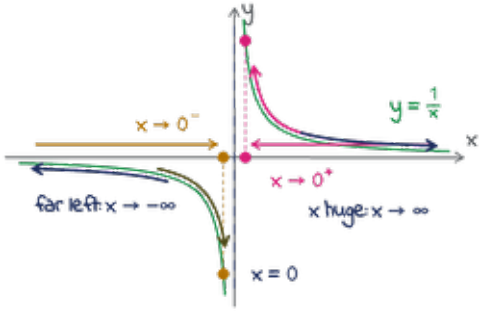
Fail 1: a jump → a name, not a reason

PICTURE 2: UP OR DOWN WITHOUT BOUND (Y = 1/X)

Picture 2: without bound

Picture 2. without bound

$y = \frac{1}{x}$



Think: $\frac{1}{0^+} \to \infty$, $\frac{1}{0^-} \to -\infty$

Think: $\frac{1}{\infty} \to 0$, $\frac{1}{-\infty} \to 0$

$0^+ \to$ a tiny positive x; 0^+, $\infty \to$ not numbers

$\frac{1}{0} \to$ undefined: never divide by 0

on the AP → write limits

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to \infty} \frac{1}{x} = 0$, $\lim_{x \to -\infty} \frac{1}{x} = 0$

0 is real → these two limits exist

Up vs down → $\lim_{x \to 0} \frac{1}{x}$ does not exist

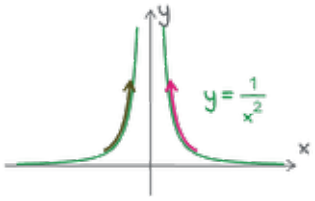
Not even $\pm\infty$ → Fail 2: unbounded

WHEN CAN WE WRITE = INFINITY? (Y = 1/X^2)

When can we write = ∞?

When can we write = ∞?

$x^2 > 0$ on both sides → $\frac{1}{x^2} > 0$ on both sides



Same way up → $\lim_{x \to 0} \frac{1}{x^2} = \infty$

$= \infty \to$ how it fails: unbounded

limit exists?

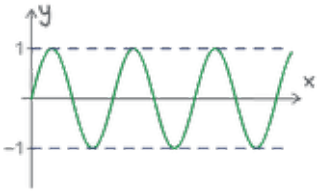
∞ not real → the limit does not exist

PICTURE 3: SWINGING FOREVER

Picture 3. Swinging forever

$y = \sin x$

as $x \to \infty$

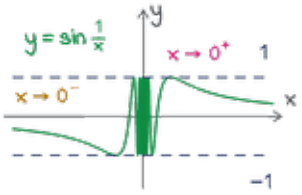


$[-1, 1]$ → settles? No trapped ≠ settled

→ $\lim_{x \to \infty} \sin x$ does not exist

Now $\sin \frac{1}{x}$, as $x \to 0$

$x \to 0^+ \to \frac{1}{x} \to \infty \to$ Think: $\sin(\infty)$



Never settles → oscillating

$\lim_{x \to 0} \sin \frac{1}{x}$

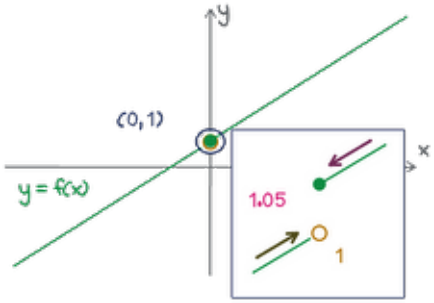
does not exist

READ WHAT IS GIVEN: A HIDDEN JUMP

Read what is given

Careful read what is given

Given → f(0) = 1.05



Left → 1, right → 1.05

$\lim_{x \to 0} f(x)$ does not exist

AP: scale can hide behavior

Doesn't match → zoom in first

one-sided off a graph → wrong side, point lost

PROBLEM 1 OF 2

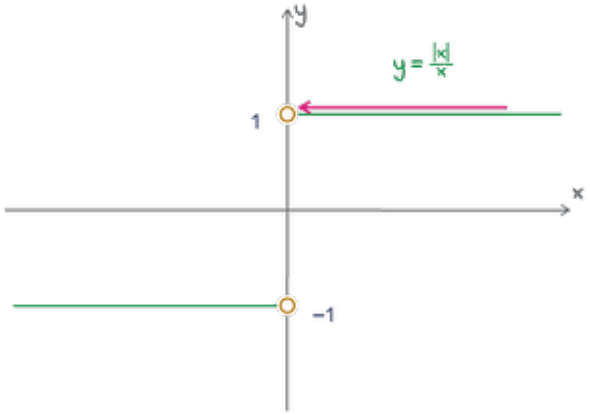
Absolute value over x

Let $f(x) = \dfrac{|x|}{x}$.

(a) Find $\lim_{x \to 0^+} f(x)$.

(b) Find $\lim_{x \to 0^-} f(x)$.

(c) Does $\lim_{x \to 0} f(x)$ exist?



Solution (a) The right side

(a)

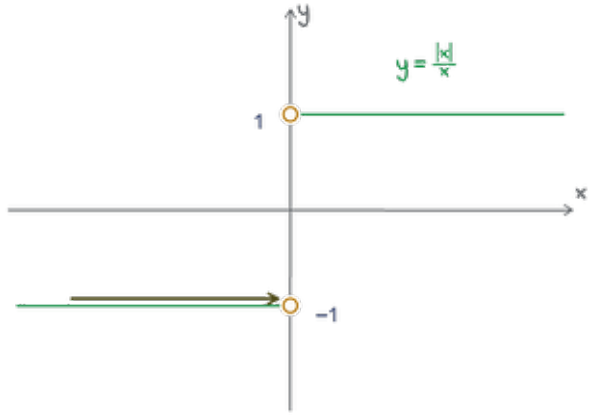
Plan: $x > 0$, $x < 0$ → each side → compare

$x > 0 \to |x| = x \to f(x) = \dfrac{x}{x} = 1$

$x \to 0^+$ → height 1

$\lim_{x \to 0^+} f(x) = 1$

Solution (b) The left side

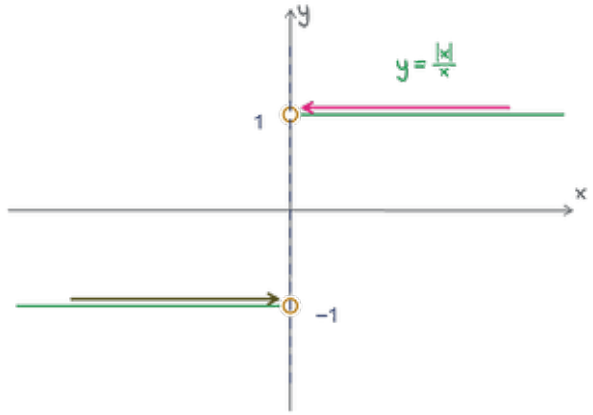


(b)

$x < 0 \to |x| = -x$, $f(x) = \dfrac{-x}{x} = -1$

$\lim_{x \to 0^-} f(x) = -1$

Solution (c) Both sides



(c)

$1 \neq -1$

$\lim_{x \to 0} f(x)$ does not exist

No point at 0 → doesn't matter

PROBLEM 2 OF 2

A jump and an asymptote

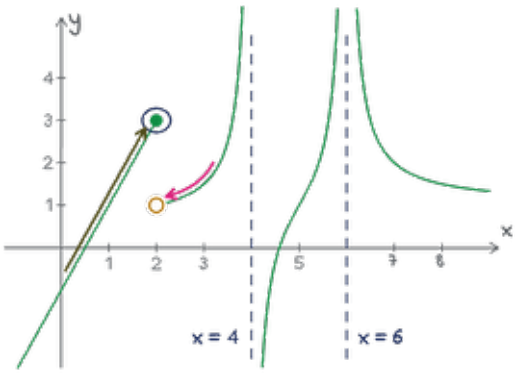
The graph of g, with vertical asymptotes $x = 4$ and $x = 6$, is shown.

(a) Find $g(2)$ and $\lim_{x \to 2} g(x)$.

(b) Find both one-sided limits at $x = 4$.

(c) Does $\lim_{x \to 4} g(x)$ exist?

(d) Find $\lim_{x \to 6} g(x)$.



Solution (a) Value, then each side

(a)

Plan: dot → value; sides → limit

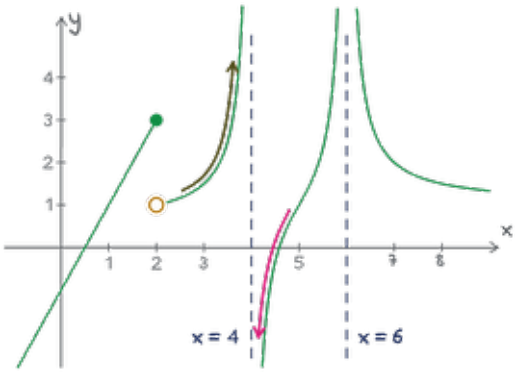
$g(2) = 3$

$x \to 2^- \to 3$

$x \to 2^+ \to$ open circle, $1 \neq 3$

$\to \lim_{x \to 2} g(x)$ does not exist

Solution (b) Each side at the asymptote



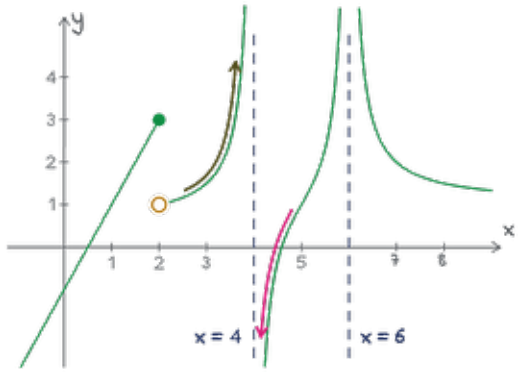
(b)

$x \to 4^- \to$ up, unbounded

$\lim_{x \to 4^-} g(x) = \infty$

$\lim_{x \to 4^+} g(x) = -\infty$

Solution (c) Opposite ways

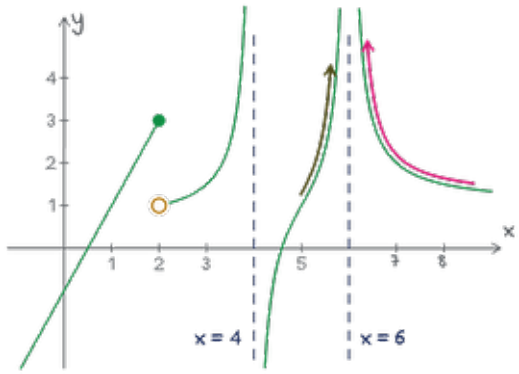


(c)

Opposite ways → not even $\pm\infty$

$\lim_{x \to 4} g(x)$ does not exist

Solution (d) The same way up



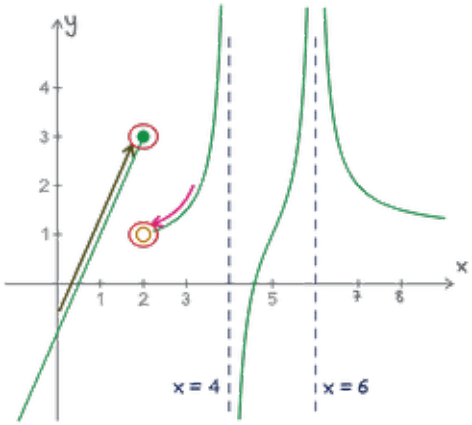
(d)

$x \to 6^-, x \to 6^+ \to$ both up

Same way → $\lim_{x \to 6} g(x) = \infty$

→ no real-number limit

Careful → where points are lost



X Common mistake

Dot, left → 3

$\lim_{x \to 2} g(x) = 3$

→ dot + one side

✓ Do this

Left → 3, right → 1

3 ≠ 1 → does not exist

WRAP-UP

When is there no limit?

Jump → both limits, not equal

Unbounded → Think, then write the limit

Oscillating → never settles

A picture can hide a jump → zoom in

Workbook → a video for each problem

Next → 1.4: table only

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.