1.3 / Limits from Graphs

Watch the lesson ➤

Estimating limit values from graphs

Learning goal
Read one-sided and two-sided limits from a graph, and tell when a limit does not exist.

Key ideas

01 $x \to a^-$ means $x < a$ near a; $x \to a^+$ means $x > a$ near a. The sign describes the input.

02 At $x = a$, follow the curve from both sides. The eye goes to the dot up at 4, but from the left the heights head to 2, and from the right to 2 as well: $\lim_{x\to 2} f(x) = \mathbf{2}$.

03 An open circle is a hole: the curve heads there, but that point is missing. The filled dot is the value: $f(2) = 4$. For the limit, $x \to 2$ means $x \neq 2$, so the dot does not count. The **value** is not the limit: the value is 4, the limit is 2.

04 For $f(x) = \dfrac{|x|}{x}$, use $|x| = x$ on the right of 0 and $|x| = -x$ on the left. The one-sided limits are 1 and -1.

05 A limit has to be one real number, and both sides have to head to that same number. At $x = 4$ the left heads to 1, the right to 3; $1 \neq 3$, so $\lim_{x\to 4} g(x)$ does not exist. The graph jumps from one height to the other: a **jump**.

06 For a constant $c > 0$, from the graph of $y = \dfrac{c}{x}$: $\lim_{x\to 0^+} \dfrac{c}{x} = \infty$, $\lim_{x\to 0^-} \dfrac{c}{x} = -\infty$, $\lim_{x\to\infty} \dfrac{c}{x} = 0$, $\lim_{x\to -\infty} \dfrac{c}{x} = 0$. The shorthand $\dfrac{c}{0^+} = \infty$, $\dfrac{c}{0^-} = -\infty$, $\dfrac{c}{\infty} = 0$ is only to feel it, **not** for the exam. Opposite ways at 0: $\lim_{x\to 0} \dfrac{c}{x}$ does not exist, not even ∞ or $-\infty$. But $\dfrac{c}{x^2} > 0$ on both sides and both sides go up: $\lim_{x\to 0} \dfrac{c}{x^2} = \infty$. $= \infty$ says how it fails: **unbounded**. ∞ is not a real number, so there is still no real-number limit.

07 Trapped between -1 and 1 is not settling: however far out, $\sin x$ still hits 1 and -1, so $\lim_{x\to\infty} \sin x$ does not exist. Near 0, $\dfrac{1}{x}$ heads to $\pm\infty$ and $\sin \dfrac{1}{x}$ swings faster and faster, never settling on one height: **oscillating**. $\lim_{x\to 0} \sin \dfrac{1}{x}$ does not exist.

08 Read what is given. Given $f(0) = 1.05$, a graph that looks like one line through $(0, 1)$ can hide a jump: zoom in, the left heads to 1, the right to 1.05, so the limit at 0 does not exist. Because of scale, a graph can miss important behavior: if the picture does not fit what is given, zoom in before you decide.

Remember

one-sided limit
A limit from one side only: $x \to a^+$ (right) or $x \to a^-$ (left).

hole
An open circle: the graph heads to that point, but the point is not on the graph.

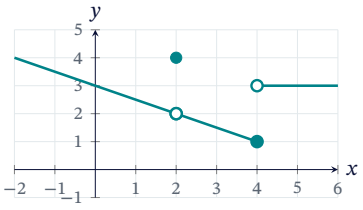
jump
A break where the left and right sides head to different heights.

unbounded
The graph grows (or falls) without bound. lim = ∞ says how a limit fails; it is not a real-number limit.

vertical asymptote
A vertical line x = c where the graph grows or falls without bound.

oscillating
The graph keeps swinging up and down and never settles on one height.

Reading a graph near a point



Near $x = 2$: the limit is 2 (the hole), and $f(2) = 4$ (the dot). At $x = 4$: left 1, right 3, so no limit.

Class notes

1.3 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write each side with its sign: $\lim_{x\to 2^-} g(x) = 3$ and $\lim_{x\to 2^+} g(x) = 1$.	Do not write $\lim_{x\to 2} g(x) = 3$ and 1. A limit is one real number.
When the sides differ, say so in words: "$\lim_{x\to 2} g(x)$ does not exist, because the one-sided limits are not equal."	Do not write $f(1) = \lim_{x\to 1} f(x)$ unless the graph shows it: read the dot and the limit separately.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The dot is not the limit

Code L2

✗ $\lim_{x\to 4} f(x) = 6$ because the filled dot is at $(4, 6)$.

✓ Follow the graph toward $x = 4$. If it heads to the open circle at height 2, then $\lim_{x\to 4} f(x) = 2$; the dot only gives $f(4) = 6$.

02 The dot and one side are not the limit

Code L3

✗ $\lim_{x\to 2} g(x) = 3$: the dot is at 3, and one side heads to 3.

✓ Read each side on its own: the left heads to 3, the right heads to 1. $3 \neq 1$, so $\lim_{x\to 2} g(x)$ does not exist.

03 Opposite infinities are not infinity

Code L12

✗ The graph shoots up next to $x = -3$, so $\lim_{x\to -3} h(x) = \infty$.

✓ Check each side. If $\lim_{x\to -3^-} h(x) = \infty$ but $\lim_{x\to -3^+} h(x) = -\infty$, the limit does not exist.

04 The side sign belongs to x

Code L8

✗ $\lim_{x\to 6^-} f(x) = 4$ means the outputs stay below 4.

✓ The minus is on the input: $x < 6$, with x close to 6. The outputs may be above or below 4.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ tests this topic directly. The CED sample multiple-choice question 11 reads a one-sided limit from a graph, in the calculator section, with a "nonexistent" choice.

1.3 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Example 01

Video · E01

Let $f(x) = \dfrac{|x|}{x}$.

(a) Find $\lim_{x \to 0+} f(x)$.

(b) Find $\lim_{x \to 0-} f(x)$.

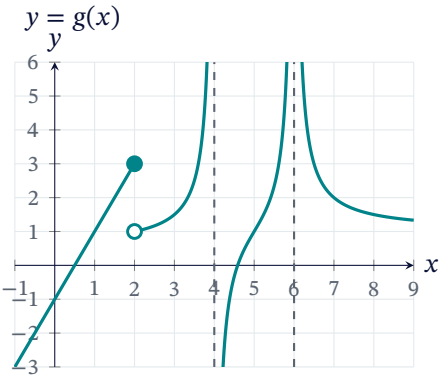
(c) Does $\lim_{x \to 0} f(x)$ exist?

> Rewrite the absolute value for positive and negative inputs.

Example 02

Video · E02

(a) Find $g(2)$ and $\lim_{x \to 2} g(x)$.
(b) Find both one-sided limits at $x = 4$.
(c) Does $\lim_{x \to 4} g(x)$ exist?
(d) Find $\lim_{x \to 6} g(x)$.



> At each point, follow each side on its own.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.3 / Example answers

Limits from Graphs

E01 (a) 1; (b) -1; (c) does not exist

E02 (a) $g(2) = 3$ (the filled dot); the limit does not exist: left 3, right 1; (b) $\lim_{x \to 4^-} g(x) = \infty$, $\lim_{x \to 4^+} g(x) = -\infty$; (c) No: the sides go opposite ways, so there is no limit, not even $\pm\infty$; (d) $\lim_{x \to 6} g(x) = \infty$: both sides rise without bound (∞ is not a real number)