1.1 / Practice workbook

Instantaneous Rate of Change

1.1 / AP-style free response

Problem 1 ➤ */ Show the work and the reasons a grader looks for.*

A rocket rises. Its height is s at time t. Consider two moments. Between them, Δt is the change in time and Δs is the change in height, with $\Delta t \neq 0$.

(a) What does $\dfrac{\Delta s}{\Delta t}$ represent?

(b) To find the rate of change at one instant, can we let $\Delta t = 0$? Explain.

(c) How, then, can we find the rate of change at one instant?

1.1 / AP-style free response

Problem 2 ➤ / *Show the work and the reasons a grader looks for.*

A runner starts at time $t = 0$. $D(t)$ is the distance, in meters, that the runner has run after t seconds. Selected values of $D(t)$ are given in the table.

t (seconds)	0	4	8	9	11	20
$D(t)$ (meters)	0	20	48	55	69	150

(a) Find the average rate of change of D from $t = 9$ to $t = 11$. Give units.

(b) To estimate the rate of change of D at $t = 10$, which is better: $\frac{150 - 0}{20 - 0} = 7.5$ or your answer to (a)? Explain.

(c) Explain the meaning of $D(11)$, of $\frac{D(11) - D(9)}{11 - 9}$, and of $\frac{D(10) - D(9.999)}{10 - 9.999}$ in context.

1.1 / Answer key

Instantaneous Rate of Change

1 (a) The slope of the line through the two points: the average rate of change of the height (the average velocity); (b) No: with $\Delta t = 0$ we would divide by zero; (c) Slide the second moment in: Δt gets close to 0, never equal to 0. The rate of change at the instant is the limit of the average rate of change

2 (a) $\dfrac{D(11) - D(9)}{11 - 9} = \dfrac{69 - 55}{2} = 7$ meters per second; (b) Part (a): its times, 9 and 11, are the closest times in the table on both sides of 10.; (c) $D(11)$: the runner has run 69 meters after 11 seconds. The quotient over $[9, 11]$: the average speed, in meters per second, from 9 to 11 seconds. The last quotient: the average speed over the last 0.001 second before $t = 10$, very close to the rate of change at the instant $t = 10$.