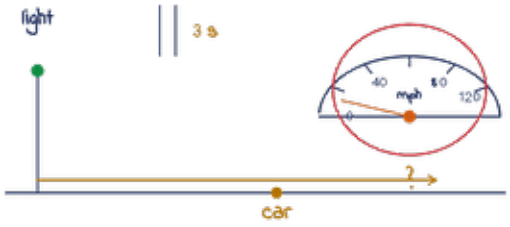
Lecture Notes · Unit 1 · Lecture 1 AP CED 1.1

Instantaneous rate of change

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

THE SPEEDOMETER QUESTION



Green light → takes off

Freeze at 3 s → one number

Speed = $\frac{\text{distance}}{\text{time}}$

Frozen frame → no time

That number = $\frac{\text{what}}{\text{what}}$?



t s → t^2 m

→ $s(t) = t^2$

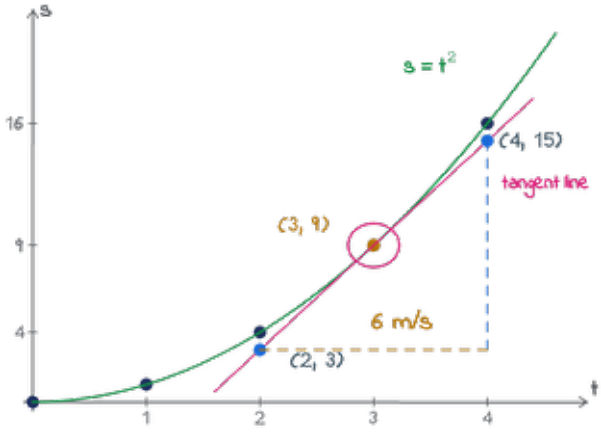
t (s)	0	1	2	3	4
s (m)	0	1	4	9	16

Gaps → 1, 3, 5, 7 m

At 3 seconds → how fast?

FROM THE ROAD TO A GRAPH

The car's position graph



Horizontal: time t (s) · vertical: position s (m)

s(3) = 9 → where the car is

$\frac{9\text{ m}}{3\text{ s}}$ = 3 m/s ?

→ average over the whole 3 s

9 → 16 m → 3 is too small

Two points, one second apart

Two points → (3, 9) and (4, 16)

Δt (time gap) = 4 − 3 = 1 s

Δs = 16 − 9 = 7 m

$\frac{\Delta s}{\Delta t} = \frac{16-9}{4-3} = 7$ m/s

Average rate of change (average velocity)

Through two points → secant line

$\frac{s(b)-s(a)}{b-a}$

Shrink the time gap

7 → still an average: too big

Shrink the time gap: 1 s → 0.5 s

$s(3.5) = 3.5^2 = 12.25$ m

$\frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$ m/s

Smaller gap → closer to the truth

Why not a time gap of zero?

Gap 0 → $\frac{9-9}{3-3} \to \frac{0}{0}$?

$\Delta t = 0$ → can't divide

Can't be 0 → but can get close to 0

Gap 0.1 → average 6.1

Gap 0.01 → average 6.01

Before 3 s: $\frac{8.41 - 9}{2.9 - 3} = \frac{-0.59}{-0.1} = 5.9$

Gap 0.01 → 5.99

Where are they heading?

After 3 s: 7, 6.5, 6.1, 6.01 → 6

Before 3 s: 5.9, 5.99 → 6

As $t \to 3$, $\frac{s(t) - 9}{t - 3} \to 6$

→ means "gets close to"

Instant → Δt (time gap) close to 0

Instantaneous rate of change: 6 m/s

Speedometer, answered

Speedometer → 6 m/s ≈ 13 mph

Not a guess → both sides, one number

Just touching the curve → tangent line

(2, 3), (4, 15): on the line, not the curve

$\frac{15 - 3}{4 - 2} = \frac{12}{2} = 6$

Only a graph? → draw the tangent, read two points

ON THE AP EXAM

"Find the average rate of change over ..."

Table → rate at one time → closest times, both sides

Use the table values

Fast way → Unit 2; what 6 means

Lost points → order, wide interval, no units; average vs. instant

PROBLEM 1 OF 1

Filling a tank

Let $W(t)$ be the number of gallons of water in a tank after t minutes.

(a) Find the average rate of change of W from $t = 5$ to $t = 7$, with units.

(b) Which is the better estimate of the rate at $t = 6$: $\frac{62 - 0}{10 - 0} = 6.2$ or your answer to (a)? Explain.

(c) Explain the meaning of $W(7)$, $\frac{W(7) - W(5)}{7 - 5}$, and $\frac{W(6) - W(5.999)}{6 - 5.999}$ in context.

t (min)	0	2	5	7	10
w (gal)	0	4	16	32	62

Solution (a) Change in W over change in t

(a)

Plan: change in w over change in t

$\frac{32 - 16}{7 - 5} = \frac{16}{2} = 8$

→ 8 gallons per minute

order mixed up: $\frac{16 - 32}{7 - 5} \to -8$?

Solution (b) Which is the better estimate?

t (min)	0	2	5	7	10
w (gal)	0	4	16	32	62

(b)

Plan: the narrowest interval around t = 6

Whole table

5 to 7 → narrowest interval around 6 → 8 is the better estimate

Solution (c) Explain the meaning of each.

t (min)	0	2	5	7	10
w (gal)	0	4	16	32	62

(c)
w(7) → 32 gal at 7 min (an amount)

$\frac{w(7)-w(5)}{7-5}$ → average rate of change of w, 5 to 7 min (gal/min)

6 − 5.999 → 0.001 min

→ average ≈ rate at the instant t = 6 (gal/min)

Workbook → same 3 questions

WRAP-UP

Back to the speedometer

Average rate of change → needs a time gap

Gap of 0 → no number. Gap close to 0 → one number.

That number → instantaneous rate of change

"Closing in on a value" → a limit

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.