1.1 / Instantaneous Rate of Change

Watch the lesson ➤

From average rates of change on an interval to the rate at one instant

Learning goal

Use average rates of change to describe change over an interval and estimate the rate at one instant.

Key ideas

01 The average rate of change is change in output divided by change in input. For position s at time t, it is $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$.

02 On a position graph, the average rate of change is the slope of the secant through the two points.

03 Move the second point closer to the first. When these secant slopes approach one number, that number is the instantaneous rate of change.

04 The instantaneous rate is the slope of the tangent. The time interval can get as close to 0 as we like while remaining nonzero.

Remember

average rate of change
Change in output per unit of input over an interval.

secant line
A line through the two points used to find an average rate of change.

instantaneous rate of change
The rate at one time, approached by average rates of change over shorter intervals.

tangent line
Its slope gives the instantaneous rate at the point.

Rate toolbox

$$\text{average rate of change} = \frac{\text{change in output}}{\text{change in input}}$$

$$\text{rate units} = \frac{\text{output units}}{\text{input units}}$$

Class notes

1.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Use the same order in both differences: $\frac{W(7)-W(5)}{7-5}$. Give the answer in gallons per minute.	A rate over $[5,7]$ gives an average. Call it an estimate when you use it for the rate at $t=6$.
To estimate a rate at a time from a table, use the closest available times on each side.	Keep the time interval nonzero.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 An instant is not a zero change in time

Code L15

✗ For the rate at one instant, put $\Delta t = 0$ in $\frac{\Delta s}{\Delta t}$.

✓ That divides by zero. Δt gets close to 0, never equal to 0: the rate at an instant is the limit of the average rate of change.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released question tests this introductory topic alone.

1.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Average rates of change and the rate at an instant	Write change in output over change in time. For an estimate at one time, take the closest times on both sides.	E01 · Practice 1, 2

Example 01

Video · E01

$W(t)$ is the number of gallons of water in a tank after t minutes.

t (min)	0	2	5	7	10
$W(t)$ (gal)	0	4	16	32	62

(a) Find the average rate of change of W from $t = 5$ to $t = 7$. Give units.

(b) Which better estimates the rate at $t = 6$: $\dfrac{62-0}{10-0} = 6.2$ or your answer to (a)? Why?

(c) Explain the meaning of $W(7)$, $\dfrac{W(7)-W(5)}{7-5}$, and $\dfrac{W(6)-W(5.999)}{6-5.999}$.

> Use two table entries for an average rate of change. For a rate at one time, compare how close the two times are.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.1 / Example answers

Instantaneous Rate of Change

E01 (a) 8 gallons per minute; (b) Part (a). The times 5 and 7 are the closest table times on either side of 6.; (c) The tank holds 32 gallons at 7 minutes. The first quotient is the average rate of change from 5 to 7 minutes. The last is the average rate of change over the 0.001 minute before 6, which estimates the instantaneous rate at 6.